April 25 - Section 3.8: Proof of the Alexander Theorem, II

April-23-12
7:37 PM

Wed May 9 → Thu May 10?

$A(K)(X) := \det\left(I + T\left(I - X^{-(s_i d_i)}\right)\right)$

Theorem with $w: x^k \longmapsto w_k$,

$Z^w(K) = \exp_{\mathcal{A}^w}\left(sl_L(K) D_L\right) \cdot \exp_{\mathcal{A}^w}\left(sl_R(K) D_R\right)$

$\cdot \exp_{\mathcal{A}^w}\left(-w\left(\log_{\mathbb{Q}[[x]]} A(K)(e^x)\right)\right)$

Trick: Use $E\Psi = (\deg \Psi)\Psi$ $\left[Ef(x) = x\frac{\partial}{\partial x}f\right]$

and $\tilde{E}\zeta = \zeta^{-1}E\zeta$.

on empty board:

RHS: ! on perturbations of the identity, $\tilde{E}$ is 1-1.

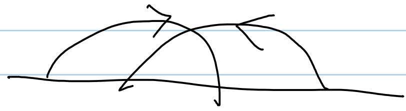
2. **Proposition 3.30.** *The following hold true:*

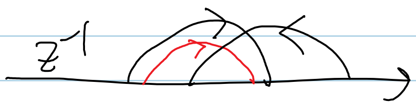
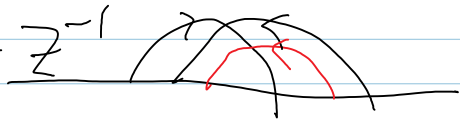
(1) *E is a derivation: $E(fg) = (Ef)g + f(Eg)$.*
(2) *If Z_1 commutes with Z_2, then $\tilde{E}(Z_1Z_2) = \tilde{E}Z_1 + \tilde{E}Z_2$.*
(3) *If z commutes with Ez, then $Ee^z = e^z(Ez)$ and $\tilde{E}e^z = Ez$.*
(4) *If $w: A \to \mathcal{A}$ is a morphism of graded algebras, then it commutes with E and $\tilde{E}$.*

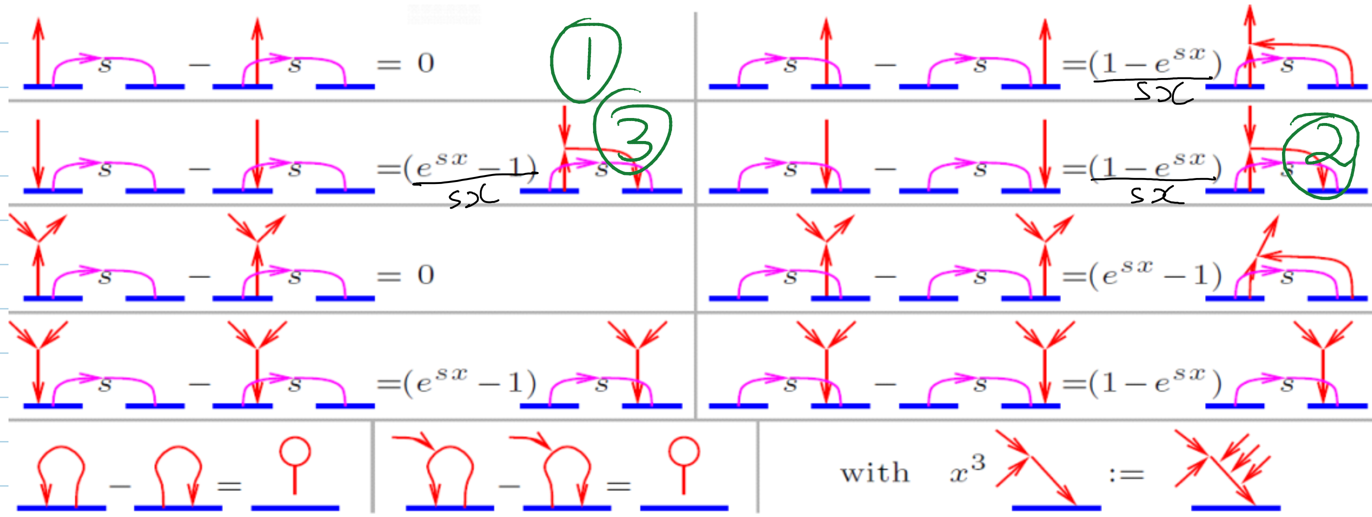
$$\tilde{E}Z_1(K) = sl_L D_L + sl_R D_R - w(E \log A(K)(e^x)) = SL - w\left(x\frac{d}{dx}\log A(K)(e^x)\right),$$

with $SL := sl_L D_L + sl_R D_R$. The rest is an exercise in matrices and differentiation. $A(K)$ is a determinant ([20]), and in general, $\frac{d}{dx}\log\det(M) = \operatorname{tr}\left(M^{-1}\frac{d}{dx}M\right)$. So with $B = T(e^{-xS} - I)$ (so $M = I - B$), we have

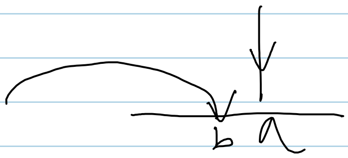
$$\tilde{E}Z_1(K) = SL + w\left(x\operatorname{tr}\left((I-B)^{-1}\frac{d}{dx}B\right)\right) = SL - w\left(x\operatorname{tr}\left((I-B)^{-1}TSe^{-xS}\right)\right),$$

LHS. Apply $\tilde{E}$ to $z =$  & get

z^{-1}  $+ z^{-1}$ 



From http://www.math.toronto.edu/~drorbn/Talks/Toronto-1005/

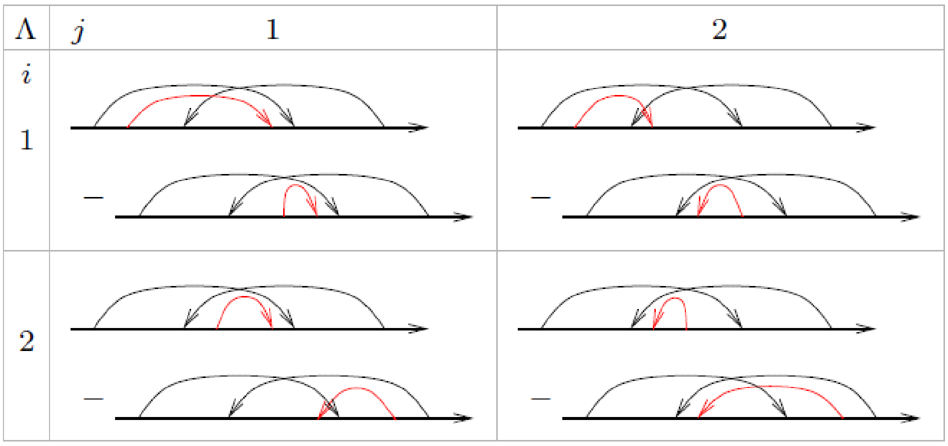


$e^{\operatorname{ad} b}(a) = e^b a e^{-b}$

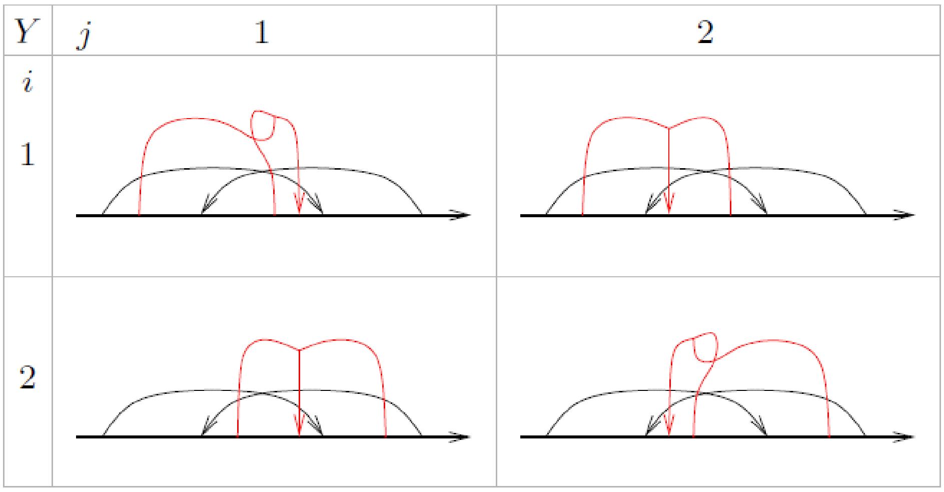
$e^x = 1 + \frac{e^x - 1}{x}x$

$$e^b a = e^{\operatorname{ad} b}(a)e^b = ae^b + \frac{e^{\operatorname{ad} b}-1}{\operatorname{ad} b}([b,a])\cdot e^b$$

The cheese-grater approach.



$\lambda_{ij} = \lambda'_{ij} - \lambda''_{ij}$



y_{ij}

Claim.
$$\begin{aligned} \lambda - SL &= \operatorname{tr} S\Lambda \\ \Lambda &= -BY - TX^{-S}w_1 \\ Y &= BY + TX^{-S}w_1 \end{aligned}$$

... Now solve for Y ...

Remark 3.38: IAM_K, λ, δ_L, δ_R, W and I don't know what in Alexander Theory this corresponds to.

Section 3.9. The relationship w/ u-knots

The diagram $K^u(\uparrow) \xrightarrow{Z^u} \mathcal{A}^u(\uparrow)$

$$\begin{tikzcd} {} \arrow[d] & {} \arrow[d] \\ K^w(\uparrow) \arrow[r, "Z^w"] & A^w(\uparrow) \end{tikzcd}$$

Commutes [by comparison w/ known results of Kricker's]. Also for round knots, though then it is silly:

$$\begin{tikzcd}
K^u(\bigcirc) \arrow[rrr, "Z"] \arrow[dddd] \arrow[dr, leftrightarrow, "\simeq"] & & & A^u(\bigcirc) \arrow[dddd, "\text{silly}"] \arrow[dl, leftrightarrow, "\simeq"'] \\
& K^u(\uparrow) \arrow[r, "1-1?"] \arrow[dd] & A^u(\uparrow) \arrow[dd, "\text{not } 1-1"] & \\
& & & \\
& K^w(\uparrow) \arrow[r, "\text{not } 1-1!"] \arrow[dl, shift left, "\text{no}"'] & A^w(\uparrow) \arrow[dr] & \\
K^u(\bigcirc) \arrow[rrr, "Z^w", "\text{a silly invt.}"'] & & & A^w(\bigcirc) = 1D \arrow[ul, dashed, bend right, "\text{no!}"']
\end{tikzcd}$$