Q1 a) $\partial^2 = 0$

$$\partial(\partial c) = 0$$
$$\|$$
$$\partial b$$

Q3 (midterm) $\underline{\omega = F_1\,dx_1 + \dots + F_3\,dx_3}$

$\underline{d\omega}$

b) find a diff. form ω: $\int_b d\omega \neq 0$ (show that $\partial b = 0$ by def)

(look at HW15)

or HW16

Q4 (midterm), $c_0, c_1 \in C_1(\mathbb{R}^2)$ $c_0 : [0,1] \to \mathbb{R}^2$

$c_1 : [0,1] \to \mathbb{R}^2$

$c_0(t) = (0,0)$

$c_1(t) = (\cos(2\pi t), \sin(2\pi t))$

WTS: $c \in C_2(\mathbb{R})$: $\partial c = c_1 - c_0$

$c : [0,1]^2 \to \mathbb{R}^2$

Typo:

$\tilde{c}_0(t) = (0,0)$

$\tilde{c}_1(t) = (\cos(t), \sin(t))$

Suppose that $\partial\tilde{c} = \tilde{c}_0 - \tilde{c}_1$

Idea: $\partial(\tilde{c}_0 - \tilde{c}_1) \neq 0$

$$c(r,t) = (r\cos(2\pi t), r\sin(2\pi t))$$

Q3: ω

I Statement: the only obstr. to the exactn. of ω

$$\int_{S^1} \omega = 0$$

$$\underline{\int_{S^1} \omega = \lambda \int_{S^1} \eta}$$

$$\int_{S^1} \omega - \lambda \eta = 0$$

$d\omega = 0$ closed

$\exists \eta : d\eta = \omega$ exact

consider

II go to polar coord., pull back $f^* \omega$

$\Gamma_f = g^{-1}(0)$ for a nice choice of

Q4 $f: \mathbb{R}^n \to \mathbb{R}^m$

$g: \mathbb{R}^{n+m} \to \mathbb{R}^m$

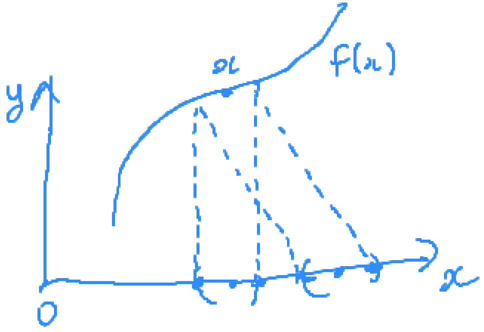
$(x, y) \longmapsto y - f(x)$

$\Gamma_f = \{(x, f(x))\} \subseteq \mathbb{R}^{n+m}$

$\{(x,y) : y = f(x)\}$

$n = m = 1$

$f : \mathbb{R} \to \mathbb{R}$



$$
\begin{tikzcd}
 & \Gamma_f \arrow[dl, "p_x"', "(x,y)\mapsto x"] \arrow[dr, "p_y", "(x,y)\mapsto y"'] & \\
\mathbb{R} & & \mathbb{R}
\end{tikzcd}
$$

attempt

$$
\begin{tikzcd}
 & \Gamma_f \arrow[dr, "(x,y)\mapsto y"] & \\
(\!-\!) \arrow[ur, "\text{“lift”}", "x \mapsto (x,f(x))"'] & & \mathbb{R}
\end{tikzcd}
$$

if this is smooth, we are good!

$f = x \longmapsto (x, f(x)) \longrightarrow f(x)$

Γ_f is a mfld (⇐)

$\exists\, U \ni x,\ W \subset \mathbb{R}$,

$h : W \to \mathbb{R}^2$

(1) $h(W) \subseteq \Gamma_f$

(2) $h'(y) > 0$

(3) $h^{-1} : f(W) \to W$ is cont.

Thm 5.2 $\longrightarrow$

$$
\begin{tikzcd}
 & \Gamma_f \arrow[dr, "p_y"] & \\
(\!-\!) \arrow[ur, "h"] & & \mathbb{R}
\end{tikzcd}
$$

QS:

Idea: locally M is diffeomorph. to a 0-meas. subset of $\mathbb{R}^{22}$.