1, 8, 11

Rk: $f \equiv \text{const} \Rightarrow m(\text{crit points}) \neq 0$, $m(\text{crit values}) = 0$

1. Thm (cov): $A \subset \mathbb{R}^n$, open, $g: A \to \mathbb{R}^n$ - bijective C^1-function

$\det g'(a) \neq 0 \quad \forall a \in A$. Then, if $f: g(A) \to \mathbb{R}$

$$\int_{g(A)} f = \int_A (f \circ g) \, |\det g'|$$

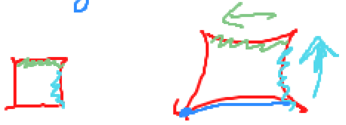
vanishes!

$$\int_{g(A)} f = \int_{\underbrace{g(A_{crit})}_{\text{crit. values}}} f + \int_{g(A^c_{crit})} f$$

zero measure

11. Idea: $d\omega = -\frac{\partial y}{\partial y} dy \wedge dx = dx \wedge dy$

(to get geom., sketch C for $f(x) = x$)



"shape" always on the left!

$$\int_C dx \wedge dy = \int_C d(-y\,dx) = -\int_{\delta C} y\,dx$$

$(0,0) \to (1,0) \to (1,1) \to (0,1) \to (0,0)$

$$\delta C = C([0,t]) \cup C([1,t]) \cup C([t,0]) \cup C([t,1])$$

$(0,t) \to (0, f(0)t)$

$(1,t) \to (1, f(1)t)$ x-coord does not change! $\int_0^1 f(t)\,dt$

$(t,0) \to (t,0)$

$(t,1) \to (t, f(t))$

$$-\int_0^1 \cancel{0}\,dx - 0 - \int_1^0 f(t)\,dt - 0$$

Q5. 1) sign (σ) = num of inversions

$$\sigma = (1\,2\,3 \dots (n-1)\,n) = \underbrace{(12)(23)(34)\dots(n-2,n-1)(n-1\,n)}_{n-1.}$$

2) $\text{sign}((i_1 j_1) \dots (i_k j_k)) = (-1)^k$.

Ex: $\boxed{\begin{array}{c}(324)\\ \shortparallel \\ 3 \to 2 \to 4 \to 3\end{array}}$

3) $\text{sign} : S_K \to \{-1, 1\}$

unique homom, $\text{sign}(12) = -1$.

$\text{sign}((ij)) = -1$.

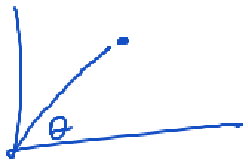
Q4: $I : \sigma \mapsto \sigma(12)$ ← involution between odd and even.

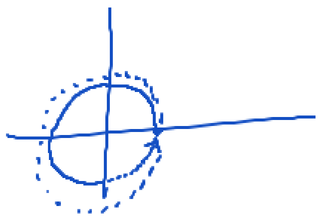
$I^2 = Id$

f^*

$$f^*(\omega) = \frac{r\cos\varphi \, d(r\sin\varphi) - r\sin\varphi \, d(r\cos\varphi)}{r^2}$$

8a) $\theta : \mathbb{R}^2 \setminus \{0\} \to [0, 2\pi)$





$$d\theta = -\frac{y}{y^2+x^2}\,dx + \frac{x}{x^2+y^2}\,dy \leftarrow$$

fails because θ is not cont.

If ω is exact, $\int_{\circlearrowleft} \omega \overset{\text{expect}}{=} 0$