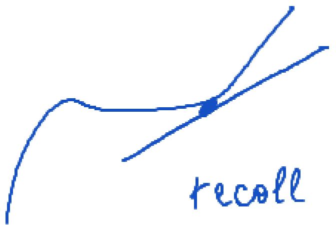
Q1 $f, g \in C^1(\mathbb{R}^n)$. (Recall: $df = \sum \frac{\partial f}{\partial x_i} dx_i$)

$$\frac{\partial}{\partial x_1}(fg) = f \cdot \frac{\partial g}{\partial x_1} + g \cdot \frac{\partial f}{\partial x_1}$$

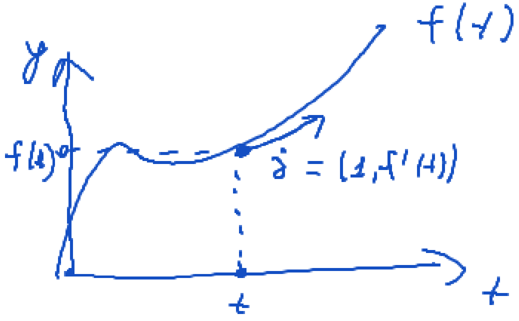
product rule!

Q2.



recall the equation of tangent line!

$$\dot{\gamma}(t) = (1, f'(t))$$



slope $(y/x) = f'$

slope of $\dot{\gamma} = f'$

$t \to \gamma(t) \to \langle \gamma(t), \gamma(t) \rangle$

chain rule!

Q3. $\|\gamma(t)\| = 1 \iff \langle \gamma(t), \gamma(t) \rangle = 1 \xrightarrow{\frac{d}{dt}} \frac{d}{dt} \langle \gamma(t), \gamma(t) \rangle = 0 = 2 \langle \dot{\gamma}(t), \gamma(t) \rangle$

$f \in C^1(\mathbb{R}^n)$.

$(\text{grad} f)(p) = \underbrace{D_1 f(p)}_{\text{f-ns from } \mathbb{R}^n \to \mathbb{R}} \cdot (e_1)_p + \ldots + \underbrace{D_n f(p)}_{\text{f-n from } \mathbb{R}^n \to \mathbb{R}} (e_n)_p$

$(e_i)_p$ form the standard basis in $T_p(\mathbb{R}^n)$

Moreover, $(e_i)_p$ form an orthonormal basis,

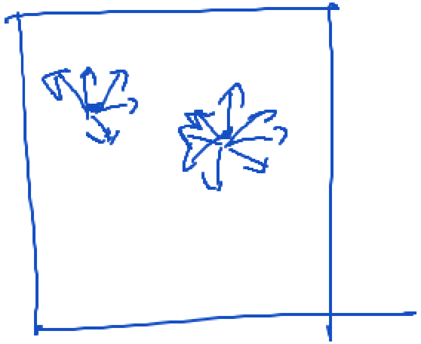
$\langle (e_i)_p, (e_j)_p \rangle = \delta_{ij}$

Ex: $v_p = e_i$ By def-n, $(e_i)_p \in T_p(\mathbb{R}^n)$

$\underbrace{D_{e_i} f = D_i f \text{ (from}}$ HW??) Hint: use HW?? to recall that $D_v + D_w = D_{v+w}$

$\langle \text{grad} f(p), e_i \rangle = \langle \sum_{m=1}^{n} D_m f(p) e_m, e_i \rangle = \langle D_i f(p) e_i, e_i \rangle = D_i f(p) \langle e_i, e_i \rangle =$

$= D_i f(p)$



$\mathbb{R}^n$ admits an affine structure

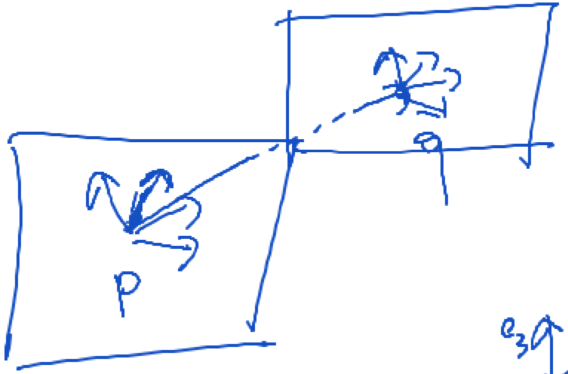
$$(p, v)$$

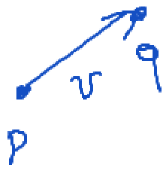
"points" "vectors"

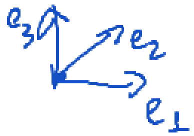
$$\lambda (p, v) = (p, \lambda v)$$

$$(p, v_1) + (p, v_2) = (p, v_1 + v_2)$$

$$p + v = q$$







$$\begin{array}{c} \tilde{X} \\ \downarrow \\ X. \end{array} \qquad \begin{array}{c} T(\mathbb{R}^n) \\ \downarrow \\ \underset{\sim}{\mathbb{R}^n} \end{array} \qquad \begin{array}{c} (p, v) \\ \downarrow \\ p \end{array}$$

fiber bundle

$\Lambda^k(V)$ $V = \mathbb{R}^n$.

$e_1 \dots e_n$

$\varphi_1 \dots \varphi_n$

$\sum_I c_I \varphi_{i_1} \wedge \dots \wedge \varphi_{i_k} \in \Lambda^k(V)$

pure algebra!

$\Lambda^n(V)$

$\int f \, dx_1 \wedge \dots dx_n \in \mathbb{R}$

$\Omega^k(\mathbb{R})$ $e_i = \frac{\partial}{\partial x_i}$ (basis vectors in $T\mathbb{R}^n$)

dx_i

$\sum_I f_I \, dx_{i_1} \wedge \dots dx_{i_k}$

$dx_i\left(\frac{\partial}{\partial x_j}\right) = \delta_{ij}$

Poincaré duality!

For a really nice shape (manifold) in $\mathbb{R}^n$

$\int : \Lambda^k(V) \longrightarrow$ n-k subshapes of the shape