polar coord. integrals
spherical coord. integrals

10. Important points:

A might be unbounded
B must be open!

Suppose A – open
$B \subset A$ is not

$A = (0,1)$
$B = \mathbb{Q}$, $B = A \setminus \mathbb{Q}$
$\chi_{\mathbb{Q}}$ – not integrable

If A, B are open, bdd, then use the int. criterion

unbounded $\to$ bounded
P.O.U.

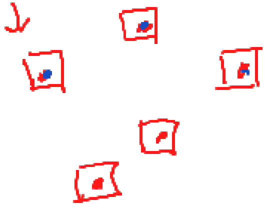
Ex: $f = e^{-x^2-y^2}$, $A = \mathbb{R}^2$

1. $\int_A f = \int_{\tilde{A}} g \rightsquigarrow \int_A f - g = 0$

illegal! bec. we haven't shown that $\int_A g$ exists.

0 everywhere except fin. coll. of points

small



values of $f-g$ are fixed, areas $\to 0$

False: we don't know if $\int_A g$ exists yet!

But: $\int_A f$ exists! $\int_A g - f$ exists and eq. to zero

$$\int_A g - f = 0 = \int_A f - g$$

$$\int_A g = \int_A f + (g-f) = \int_A f + \int_A g-f.$$

2b) $\mathbb{Q} \subset \mathbb{R}$ $\forall \varepsilon > 0$ $\exists R_1, \ldots R_n$: $A \subset \bigcup_i R_i$

2a) Assume that A is closed

3. {set of discont. of f} is of meas. 0

Show the set of discon. of $g \circ f$ is also of meas. 0.

$\frac{1}{257} \rightsquigarrow \varepsilon > 0.$

$$\bigcup_{R\in\mathcal{R}'} R \subset A \subset \bigcup_{R\in\mathcal{R}} R$$

$$\underbrace{\int_{R\in\mathcal{R}'} 1}_{\text{lower Riemann sum}} \le \int_A 1 \le \underbrace{\int_{R\in\mathcal{R}} 1}_{\text{upper Riemann sum}}$$

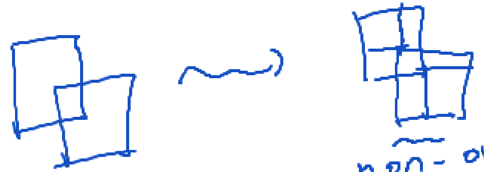
1_A - char. fn-n.

A is Jordan-meas $\Rightarrow$ 1_A is integrable.

There are part. P_L, P_U

$$\underbrace{|L(1_A, P_L) - U(1_A, P_U)| < \frac{\varepsilon}{2}}$$

1. consider the ref. of P_L, P_U

2. 

overlapp $\rightsquigarrow$ non-overlapp

result. part. by P

$$|L(1_A, P) - U(1_A, P)| < \varepsilon/2.$$

expand the sums, ident. the rectangles not in the inter. of A.

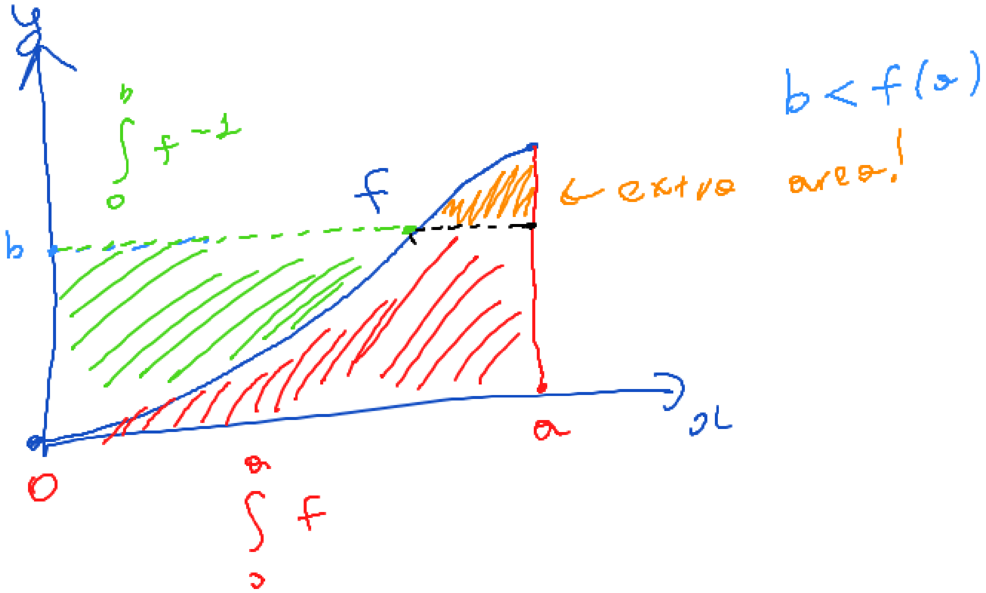
5b) Q

6) If f is not ident. zero, $\Rightarrow \exists x : f(x) = a > 0 \Rightarrow$

$\Rightarrow \exists$ open nbhd $U_x \ni x$: $\underbrace{f|_{U_x} > \frac{a}{2}}$

forces the int. to be greater than $\nu(U_x) \cdot \frac{a}{2}$.

7.



8. Fubini

11. 1. Rescale

$2x^2 + 3y^2 + 5z^2 \leq 1$

$x^2 + y^2 + z^2 \leq 1$

Answer will be a multiple of $\frac{4}{3}\pi R^3$.

12. FTC.

13. Change of Var's: $\det(\ldots) = 1$

Polar coord. $\varphi \rightsquigarrow \varphi + \theta$

14. f supp $= U$ $\quad$ $\operatorname{supp}(fg) \subseteq \underbrace{U \cap V}$

g supp $= V$

$\sum \phi_i \psi_j$ locally fin equals 1.

$\parallel$

$\sum \phi_i \sum \psi_j$

$\quad\; \shortparallel \qquad\; \shortparallel$

$\quad\; 1 \qquad\; 1$

$f : \mathbb{R}^2 \to \mathbb{R}^2$

$f = \begin{pmatrix} f_1 \\ f_2 \end{pmatrix}$

naive

$$\begin{tikzcd}
 & \begin{pmatrix} x_1 \\ F_2(x_1, x_2) \end{pmatrix} \arrow[dr, Rightarrow] & \\
\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \arrow[ur] \arrow[dr] & & \begin{pmatrix} F_1(x_1, x_2) \\ F_2(x_1, x_2) \end{pmatrix} \\
 & \begin{pmatrix} F_1(x_1, x_2) \\ x_2 \end{pmatrix} \arrow[ur] &
\end{tikzcd}$$

You have to recover x_2 from x_1 and $F_2(x_1, x_2)$
(this is the Implicit FT)

we need $\underline{\dfrac{\partial F_2}{\partial x_2} \neq 0}$ we only know that $\det J \neq 0$.

~~on~~ step i we choose an index j : $\dfrac{\partial F_i}{\partial x_j} \neq 0$.