This is a preview of what students will see when they are submitting the assignment. Interactive features are disabled.

Homework Assignment 14

Due: Monday February 22, 2021 11:59 PM (Eastern Standard Time)

Assignment description

Solve and submit your solutions of the following problems. Note also that the late policy remains strict - you will lose 5% for each hour that you are late. In other words, please submit on time!

Submit your assignment

Help

After you have completed the assignment, please save, scan, or take photos of your work and upload your files to the questions below. Crowdmark accepts PDF, JPG, and PNG file formats.

Q1 (10 points)

Spivak's 4-13b. If $f, g \colon \mathbb{R}^n \to \mathbb{R}$ are differentiable, show that $d(fg) = fdg + gdf$.

(Note. This question will make sense after Wednesday's class).

Q2 (10 points)

Spivak's 4-15. Let $f \colon \mathbb{R} \to \mathbb{R}$ be differentiable and define $\gamma \colon \mathbb{R} \to \mathbb{R}^2$ by $\gamma(t) = (t, f(t))$. Show that the end point of the tangent vector of γ at t lies on the tangent line of f at $(t, f(t))$. (Assuming the MAT157 definition of "tangent line").

Q3 (10 points)

Spivak's 4-16. Let $\gamma \colon [0,1] \to \mathbb{R}^2$ be a differentiable curve such that $|\gamma(t)| = 1$ for all t. Show that the tangent vector to $\gamma(t)$ at t is perpendicular to $\gamma(t)_{\gamma(t)}$.

Q4 (10 points)

Spivak's 4-18 (modified). If $f \colon \mathbb{R}^n \to \mathbb{R}$ is differentiable, define a vector field $\operatorname{grad} f$ by

$$(\operatorname{grad} f)(p) = D_1 f(p)(e_1)_p + \ldots + D_n f(p)(e_n)_p.$$

If v_p is some other tangent vector at p, prove that $D_{v_p} f = \langle (\operatorname{grad} f)(p), v_p \rangle$ and therefore $(\operatorname{grad} f)(p)$ is the direction in which f is changing the fastest at p.