Homework Assignment 12

Due: Wednesday February 3, 2021 11:59 PM (Eastern Standard Time)

Assignment description

Solve and submit your solutions of the following problems. Note also that the late policy remains strict - you will lose 5% for each hour that you are late. In other words, please submit on time! **Note that the questions in this assignment have unequal weights!**

Submit your assignment

Help

After you have completed the assignment, please save, scan, or take photos of your work and upload your files to the questions below. Crowdmark accepts PDF, JPG, and PNG file formats.

Q1 (10 points)

Munkres' 27-1. Which of the following are alternating tensors in $\mathbb{R}^4$?

$$f(x,y) = x_1y_2 - x_2y_1 + x_1y_1$$

$$g(x,y) = x_1y_3 - x_3y_2$$

$$h(x,y) = (x_1)^3(y_2)^3 - (x_2)^3(y_1)^3$$

Q2 (10 points)

The determinant, as a function of a list of column vectors, is alternating. Write it in terms of the elementary alternating functions ω_I (assuming the standard basis of $\mathbb{R}^n$).

Q3 (15 points)

Munkres' 28-6 (modified). Let $L\colon \mathbb{R}^m \to \mathbb{R}^n$ be a linear transformation presented by a matrix $A \in M_{n\times m}(\mathbb{R})$ relative to the standard bases of the spaces involved. If ω_I (where $I \in \underline{n}^k_a$) is an elementary alternating k-tensor on $\mathbb{R}^n$, then $L^*\omega_I$ is a linear combination $\sum_{J\in\underline{m}^k_a} c_J\omega_J$ of the elementary alternating k-tensors ω_J on $\mathbb{R}^m$. Write formulas for the coefficients c_J in terms of the matrix A.

Q4 (20 points)

Along the lines of our development of a theory of "tensors" and a theory of "alternating tensors", develop a theory of "symmetric tensors" $S^k(V)$ (a symmetric tensor is a tensor whose values are unchanged if its arguments are permuted). Your theory should include definitions for specific tensors σ_I for $I \in \underline{n}^k_s$ (what should $\underline{n}^k_s$ be)?), a proof that the σ_I indeed belong to $S^k(V)$ and that they form a basis of that space, and a computation of the dimension of $S^k(V)$.