Homework Assignment 2

Due: Wednesday September 30, 2020 11:59 PM (Eastern Daylight Time)

Assignment description

Solve and submit your solutions of the following questions. They are all taken from Spivak's book, pages 10,13, and 14.

Submit your assignment

Help

After you have completed the assignment, please save, scan, or take photos of your work and upload your files to the questions below. Crowdmark accepts PDF, JPG, and PNG file formats.

Q1 (10 points)

Spivak's 1-16. Find the interior, exterior, and boundary of the sets

$$A_1 = \{x \in \mathbb{R}^n : |x| \leq 1\},$$

$$A_2 = \{x \in \mathbb{R}^n : |x| = 1\},$$

$$A_3 = \{x \in \mathbb{R}^n : \forall i \, x_i \in \mathbb{Q}\}.$$

Q2 (10 points)

Spivak's 1-21. (a) If A is closed and $x \notin A$, prove that there is a number $d > 0$ such that $|y - x| \geq d$ for all $y \in A$.

(b) If A is closed, B is compact and $A \cap B = \emptyset$, prove that there is $d > 0$ such that $|y - x| \geq d$ for all $x \in A$ and $y \in B$ (hint available in textbook).

(c) Give a counterexample in $\mathbb{R}^2$ if A and B are closed but neither is compact.

Q3 (10 points)

Spivak's 1-22. If U is open and $C \subset U$ is compact, show that there is a compact set $D \subset U$ whose interior contains C.

Q4 (10 points)

Spivak's 1-25. Prove that a linear transformation $T : \mathbb{R}^n \to \mathbb{R}^m$ is continuos (hint available in textbook).

Q5 (10 points)

Spivak's 1-26, rephrased. Let $A = \{(x, y) \in \mathbb{R}^2 : x > 0 \text{ and } 0 < y < x^2\}$. Let $f = 1_A : \mathbb{R}^2 \to \mathbb{R}$ be the *indicator* function of A, defined by $f(x, y) = 1$ if $(x, y) \in A$, and $f(x, y) = 0$ otherwise. Show that f is not continuous at $(0, 0)$, yet its restriction to every straight line through $(0, 0)$ is continuous at $(0, 0)$.

Q6 (10 points)

Spivak's 1-28. If $A \subset \mathbb{R}^n$ is not closed, show that there is a continuous function $f : A \to \mathbb{R}$ which is unbounded (hint available in textbook).