18-327 on Tuesday October 16, hour 18: quotients, connectedness

September-11-10 12:29 PM

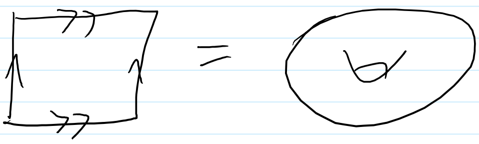
Today: (Above)

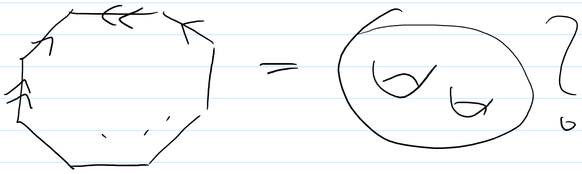
TT info Today, 6-8 SF3202 bring student ID!
Integrity rules fully enforced!
TTs are often wake-up calls.

HW4 returned

Quotients. Given a surjection $\pi: X \to Y$, equiv. $\pi: X \to X/\sim = Y$ and topology $\mathcal{T}_X$ on X, set $\mathcal{T}_Y = \{V \subset Y : \pi^{-1}(V) \in \mathcal{T}_X\}$

__Examples__ 1. Good: $S^1 = [0,1]/_{0\sim 1} = \mathbb{R}/\mathbb{Z}$

2. 

__Q__ 

3. Yet $M_{2\times 2}(\mathbb{C})/_{\text{conjugation}} = \mathbb{C}^2/_{(\lambda_1,\lambda_2)\sim(\lambda_2,\lambda_1)} \dot\cup \mathbb{C}$ is not T_2!

Connectedness. Separation, connectedness, clopen sets.

The I.V.T. If X is connected, $f: X \to \mathbb{R}$ cont., $f(x_0)<0$, $f(x_1)>0 \Rightarrow \exists x$ s.t. $f(x)=0$.

Theorem $I=[0,1]$ is connected.

Proof. Assume $0 \in A \subset I$ is clopen. Let $G = \{x : [0,x] \subset A\}$ $g = \sup G$

1. $g>0$ 2. $g \not< 1$ 3. $1 \in G$.

Theorem. A continuous image of a connected set is connected.

Theorem. If $A_\alpha \subset X$ are connected, $\bigcap A_\alpha \neq \emptyset$, then $\bigcup A_\alpha$ is connected.

Theorem. $A \subset \mathbb{R}$ is connected iff it is an interval, or a ray, or the whole thing. [I.e., if it is "convex"]

Theorem. If A is connected & $A \subset B \subset \bar{A}$, B is too.

Pf Assume C is clopen in B, $C \cap A \neq \emptyset$. Then $C \supset A$ so $Cl_X C \supset \bar{A} \supset B$,
so $Cl_X C \cap B = B$, so $Cl_B C = B$, so $C = B$.

Theorem. If $\forall \alpha$ X_α is connected, then $\prod X_\alpha$ is connected.

Example. $\mathbb{R}^\omega = \left\{ \begin{matrix} \text{bndd} \\ \text{seqs} \end{matrix} \right\} \cup \left\{ \begin{matrix} \text{unbndd} \\ \text{seqs.} \end{matrix} \right\}$ is a box-separation.