18-327 on Tuesday December 4, hour 36: Compactness in Metric Spaces (2)

September-11-10 12:29 PM

Today: Above Read: M 43, 45.

Reminders:

- Tutorials on Thursday as if it is a Monday!
- HW9 is due now, or on Thursday's tutorials, on Thu 2:30-3:30 at Bahen 6178.
- Final exam: Wed Dec 19 7-10PM GB405.

Theorem. The Following are Equivalent for a Metric X:

1. X is compact.
2. X is limit-point-compact.
3. X is sequentialy compact.
4. X is totally bounded & satisfies Lebesgue's Lemma.
5. X is totally bounded & "Complete" NTD

NTS

totally bounded $\Updownarrow$ bounded in $\mathbb{R}^n_{std}$; "Complete" $\Updownarrow$ closed NTS

Def "Complete": Every cauchy seq. converges.

Prop $X \subset \mathbb{R}^n$ is complete $\Longleftrightarrow$ X is closed.

Thm $3 \Rightarrow 5$ (Easy)

Thm $5 \Rightarrow 3$

Aside: Every metric space has a "Completion": A complete metric space in which it embeds isometrically and densely.

All done!