# 18-327 on Thursday September 20, hours 7-8: Products, subspaces

September-11-10 12:29 PM

Remember blackboard shots!

Today: Class photo at break! [Not mandatory - Just for Fun]
Products, subspaces, closed sets

Read Along: Munkres 15-17, Preread 18-19

HW1 due at 4PM, HW2 on web by midnight. !

**"The product topology"**

Given $X, Y$ topological spaces, we $\exists!$ a topology on $X\times Y$ s.t.

1. $X\times Y \xrightarrow{\pi_X} X$, $X\times Y \xrightarrow{\pi_Y} Y$ are cont. $\Big[\Longrightarrow$ If $U\subset X$ & $V\subset Y$ are open, $U\times V = \pi_X^{-1}(U)\cap\pi_Y^{-1}(V)$ must be too$\Big]$

2. $f, g: Z\to X, Y$ cont. $\Rightarrow f\times g: Z\to X\times Y$ is cont.

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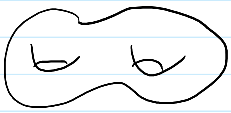
What is it all good for?

It is an extremely elegant infrastructure/language course

$\mathbb{R}^n$, , $8$, $C^\infty(\mathbb{R})$, $\mathbb{Q}_p$ ...

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Take $\mathcal{B} = \{U\times V : \begin{smallmatrix}U\subset X\\ V\subset Y\end{smallmatrix} \text{ are open}\}$

<u>Claim</u> 1 & 2 hold.

<u>Claim</u> If $\mathcal{T}_1$ & $\mathcal{T}_2$ on $X\times Y$ satisfy 1&2, then $\mathcal{T}_1 = \mathcal{T}_2$.

<u>Claim</u> $X \cong X\times\{y_0\}$ & $Y \cong \{x_0\}\times Y$.

**The Subspace Topology.** Given a T.S. $X$ and a subset $Y\subset X$, we seek a topology on $Y$ s.t.

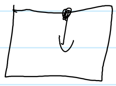
1. $i_Y: Y \hookrightarrow X$ is cont.
2. Given $f: Z\to Y \xhookrightarrow{i_Y} X$, if $i_Y\circ f$ is cont., then so is $f$.

<u>Thm</u> Such a topology exists and is unique.

Examples 1. $[0,1]$ is open in $[0,1]$
2. $[0,1)$ is open in $[0,1]$
3. "open in open is open"

**Compatibilities.** $\underbrace{\text{Sub \& Sub}}_{\text{prove}}$; $\underbrace{\text{sub \& product}}_{\text{Leave as HW}}$; bases & there, sub & order (in the convex case)

Example The $I^2_{dict}$ is different from $I^2_{ind} \subset \mathbb{R}^2_{dict}$.



not open open

**Closed sets.** Definition, 3 basic properties. closed in a subspace

closed in closed is closed

Closure & interior.

Also: $f$ is cont iff $f^{-1}(\text{closed})$ is closed.

done line

**Proposition.** $x \in \bar{A}$ iff every (basic) nbd of $x$ intersects $A$.

Definition Limit pt: $x \in A' \Leftrightarrow x \in \overline{A - \{x\}}$, iff every nbd of $x$ contains a point of $A$ other than $x$ itself.

Thm $\bar{A} = A \cup A'$

**Hausdorff spaces.** Definition.

Properties. In a $T_2$ space $X$:

1. Points are closed.
2. $x \in A'$ iff every nbd contains infinitely many points of $x$.
3. A sequence converges to at most one limit.
4. Products of $T_2$ & subspaces of $T_2$ are $T_2$.