18-327 on Thursday October 4, hours 13-14: Infinite Products, metric topologies

September-11-10 12:29 PM

Today: Boxes, cylinders, metrics

Read Alon: M19-20; Preread 21-22.

HW2/3/4 returned/due/available as usual.

New on web: Some solutions, comments by Davis (TA).

On $X = \prod_{\alpha\in I} X_\alpha = \{(x_\alpha) : x_\alpha \in X_\alpha\} = \{x : I \to \bigcup X_\alpha : x(\alpha) \in X_\alpha\}$

$\mathcal{B}_{box} = \{\prod U_\alpha : U_\alpha \subset X_\alpha \text{ is open}\}$

$\mathcal{B}_{cyl} = \{\pi_{\alpha_1}^{-1}(U_{\alpha_1}) \cap \dots \cap \pi_{\alpha_n}^{-1}(U_{\alpha_n})\}$

$= \{\prod U_\alpha : U_\alpha \subset X_\alpha$ is open, For all but finitely many α's, $U_\alpha = X_\alpha\}$

review! Note $\mathcal{B}_{box} \supset \mathcal{B}_{cyl}$.

<u>Thm</u> In both topologies:

1. Compatability w/ subspaces.

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2. If X_α is T_2 $\forall\alpha$, then $\prod X_\alpha$ is T_2.
3. $\overline{\prod A_\alpha} = \prod \overline{A_\alpha}$

<u>PF3</u> $x \in \overline{\prod A_\alpha} \iff$ If for every α U_α is a nbd of x_α, $\prod U_\alpha \cap \prod A_\alpha \neq \emptyset$

(and almost always $U_\alpha = X_\alpha$)

$\iff$ If for every α U_α is a nbd of x, $\forall\alpha$ $U_\alpha \cap A_\alpha \neq \emptyset \iff x_\alpha \in \overline{A_\alpha} \iff x \in \prod \overline{A_\alpha}$

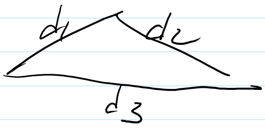
Metrics & the metric topology.

* General defs.
* Thm In a metric/metrizable space, closure = seq. closure. } not proven.
* Thm $\mathbb{R}^{\mathbb{N}}_{box}$ is not metrizable. (No seq of positive seqs goes to $\bar{0}$) done online
* Thm $\mathbb{R}^{\mathbb{R}}_{cyl}$ is not metrizable

$A=\{f:\mathbb{R}\to\mathbb{R};\ f=0$ except in finitely many places$\}$ Then $T\in \mathrm{cl}A$
$T\notin \mathrm{Seqcl}A$.

* A countable product of metrizable spaces is metrizable.
  * $\bar{d}(x,y)=\min(1,d(x,y))$

$\bar{d}$ is a metric?



if $d_1+d_2\le 1$ easy
if $d_1+d_2\ge 1$, also easy

$\bar{d}$ defines the same topology. [Two bases $\mathcal{B},\mathcal{B}'$ define the same topology iff ...]

Given (X_n,d_n) w/ d_n bndd by 1.

Define
$$d(x,y)=\sup_n \frac{1}{n}d_n(x_n,y_n)$$

* This is a metric? $[\sup(a_n)+\sup(b_n)\ge\sup(a_n+b_n)]$
* It defines the product topology?