18-327 on Thursday October 18, hours 19-20: connectedness

September-11-10 12:29 PM

Today: (Above)

TT: No comments. HW4 returned

Connectedness. Separation, connectedness, clopen sets. } should have explicitly } excluded $\emptyset$ as connected.

The I.V.T. If X is connected, $f: X \to \mathbb{R}$ cont., $f(x_0)<0$, $f(x_1)>0 \Rightarrow \exists x$ s.t. $f(x)=0$.

Theorem $I=[0,1]$ is connected.

Proof. Assume $0 \in A \subset I$ is clopen. Let $G=\{x : [0,x] \subset A\}$ $g=\sup G$

1. $g>0$ 2. $g \not< 1$ 3. $1 \in G$.

Theorem. A continuous image of a connected set is connected.

Theorem. If $A_\alpha \subset X$ are connected, $\bigcap A_\alpha \neq \emptyset$, then $\bigcup A_\alpha$ is connected.

Theorem. $A \subset \mathbb{R}$ is connected iff it is an interval, or a ray, or the whole thing. [I.e., if it is "convex"]

Thm. If X & Y are connected, then so is $X \times Y$.
(Also, if $X \times Y$ is connected & $X, Y \neq \emptyset$, then X & Y are connected)

Example. $\mathbb{R}^\omega = \left\{\begin{matrix}\text{bndd} \\ \text{seqs}\end{matrix}\right\} \cup \left\{\begin{matrix}\text{unbndd} \\ \text{seqs.}\end{matrix}\right\}$ is a box-separation.

Lemma. If A is connected & $A \subset B \subset \bar{A}$, B is too.

Pf Assume C is clopen in B, $C \cap A \neq \emptyset$. Then $C \supset A$ so $cl_X C \supset \bar{A} \supset B$, so $cl_X C \cap B = B$, so $cl_B C = B$, so $C=B$.

Theorem. If $\forall \alpha$ X_α is connected, then $\prod X_\alpha$ is connected.

Def. Path-connected; path-connected $\Rightarrow$ connected
(1. Proof from defs. } not done
2. Lemma: If X is connected, so is $f(X)$)

done line.

The topologist's sine curve

A product of path connected spaces is path-connected.