

Pensieve header: Khovanov Homology, Day 4.

Topics (in no particular order). Whatever you may suggest; whatever comes to my mind; ~~the Fibonacci numbers; the Catalan numbers; the Jones polynomial; a more efficient Jones algorithm; a riddle on spheres;~~ Khovanov homology; Γ -calculus; the Hopf fibration; Hilbert's 13th problem; non-commutative Gaussian elimination; free Lie algebras; the Baker-Campbell-Hausdorff formula; wacky numbers; ~~an order 4 torus;~~ the Schwarz Lantern; knot colourings; the Temperley-Lieb pairing; the dodecahedral link; ~~sound experiments;~~ barycentric subdivisions; ~~some Peano curves;~~ braid closures and Vogel's algorithm; ~~the insolubility of the quintic;~~ phase portraits; the Mandelbrot set; shadows of the Cantor square; quilt plots; some image transformations; De Bruijn graphs; the Riemann series theorem; finite type invariants and the Willerton fish; ~~the Towers of Hanoi; Hochschild homology of (some) coalgebras; convolutions and image improvements; the 8-5-3 milk jug problem; a cow problem;~~ a permutations package.

Outstanding Challenges

The last day to submit projects for marks will be the last day of the UofT examination period, December 20 2017 at midnight.

- Update the NCGE program to contain "backtracking information". Use it to find how to turn the lower face of a Rubik's cube by turning all but the lower face of that cube.
- Complete the package perm, with documentation and all. For Perm[5,2,3,1,4], etc, your package should know $\sigma \circ \tau$, σ^{-1} , $\sigma[[i]]$, Pivot[σ], PermutationQ[σ], IdentityPermutation[n], it should interact well with Cycles, and its internals should be hidden. It should live in a file "Perm.m".
- Write a sophisticated Mathematica package that implements QuiltPlot and DoubleQuiltPlot with many bells and whistles, as explained on November 20th.

Draw approximations of the Cantor square C^2 . Then rotate C^2 by an angle θ and project the rotation vertically, draw that projection (while explaining why it is what you think it is), and compute its measure. Perhaps also use Manipulate[..., { θ , 0, $\pi/2$ }]?

- Everybody knows that the length of a smooth curve is the supremum of the lengths of its polygonal approximations. Everybody should know that the same is not true for smooth surfaces. Can you draw a smooth surface in \mathbb{R}^3 along with a triangulated approximation thereof whose area is vastly more? An appropriate animation may be even more convincing.

Khovanov Homology

Challenge: Implement Khovanov Homology as it is defined in the handout. Your implementation should fit in one page and should be good enough to show that $\text{Kh}(5_1) \neq \text{Kh}(10_{132})$.

Khovanov: $K(L)$ is a chain complex of graded \mathbb{Z} -modules;

$$V = \text{span}\langle v_+, v_- \rangle; \quad \deg v_{\pm} = \pm 1; \quad q\dim V = q + q^{-1};$$

$$K(\bigcirc^k) = V^{\otimes k}; \quad K(\times) = \text{Flatten} \left(0 \rightarrow \underset{\text{height } 0}{K(\bigcirc)\{1\}} \rightarrow \underset{\text{height } 1}{K(\times)\{2\}} \rightarrow 0 \right);$$

$$K(\times) = \text{Flatten} \left(0 \rightarrow \underset{\text{height } -1}{K(\times)\{-2\}} \rightarrow \underset{\text{height } 0}{K(\bigcirc)\{-1\}} \rightarrow 0 \right);$$

$$\left(\bigcirc \bigcirc \xrightarrow{m} \text{cup} \right) \rightarrow (V \otimes V \xrightarrow{m} V) \quad m : \begin{cases} v_+ \otimes v_- \mapsto v_- & v_+ \otimes v_+ \mapsto v_+ \\ v_- \otimes v_+ \mapsto v_- & v_- \otimes v_- \mapsto 0 \end{cases}$$

$$\left(\text{cup} \xrightarrow{\Delta} \bigcirc \bigcirc \right) \rightarrow (V \xrightarrow{\Delta} V \otimes V) \quad \Delta : \begin{cases} v_+ \mapsto v_+ \otimes v_- + v_- \otimes v_+ \\ v_- \mapsto v_- \otimes v_- \end{cases}$$

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