Pensieve header: Khovanov Homology, Day 2.

Topics (in no particular order). Whatever you may suggest; whatever comes to my mind; ~~the Fibonacci numbers~~; ~~the Catalan numbers~~; ~~the Jones polynomial~~; ~~a more efficient Jones algorithm~~; ~~a riddle on spheres~~; Khovanov homology; Γ-calculus; the Hopf fibration; Hilbert's 13th problem; non-commutative Gaussian elimination; free Lie algebras; the Baker-Campbell-Hausdorff formula; wacky numbers; ~~an order 4 torus~~; the Schwarz Lantern; knot colourings; the Temperley-Lieb pairing; the dodecahedral link; ~~sound experiments~~; barycentric subdivisions; ~~some Peano curves~~; braid closures and Vogel's algorithm; ~~the insolubility of the quintic~~; phase portraits; the Mandelbrot set; shadows of the Cantor square; quilt plots; some image transformations; De Bruijn graphs; the Riemann series theorem; finite type invariants and the Willerton fish; ~~the Towers of Hanoi~~; ~~Hochschild homology of (some) coalgebras~~; ~~convolutions and image improvements~~; ~~the 8-5-3 milk jug problem~~; ~~a cow problem,~~ a permutations package.

An NCGE Challenge

Update the NCGE program to contain "backtracking information". Use it to find how to turn the lower face of a Rubik's cube by turning all but the lower face of that cube.

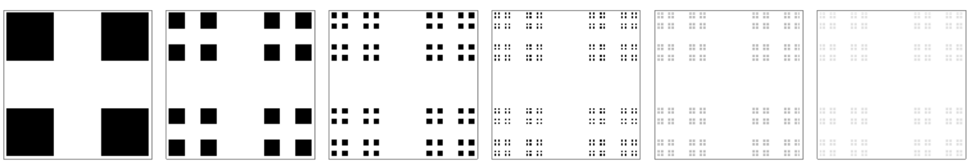
The Package Perm

Complete the package perm, with documentation and all. For Perm[5,2,3,1,4], etc, your package should know $\sigma\circ\tau$, σ^{-1}, $\sigma[\![i]\!]$, Pivot[σ], PermutationQ[σ], IdentityPermutation[n], it should interact well with Cycles, and its internals should be hidden. It should live in a file "Perm.m".

Quilt Plots

Write a sophisticated Mathematica package that implements QuiltPlot and DoubleQuiltPlot with many bells and whistles, as explained on November 20th.

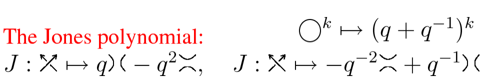
Shadows of the Cantor Square



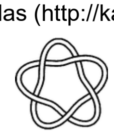
Draw the above approximations of the Cantor square C^2. Then rotate C^2 by an angle θ and project the rotation vertically, draw that projection (while explaining why it is what you think it is), and compute its measure. Perhaps also use Manipulate[..., $\{\theta, 0, \pi/2\}$]?

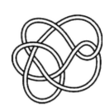
The Jones Polynomial

From the Khovanov Homology handout and from the Knot Atlas (http://katlas.org/):




3_1


5_1


10_132

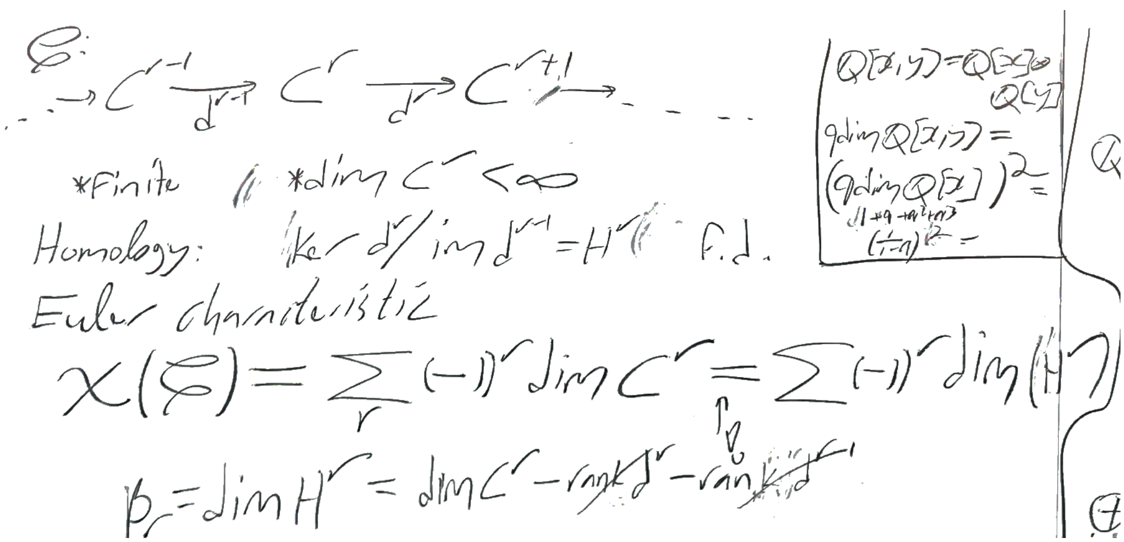
```
Jones[K_PD] := Module[{n = Length[K], P, t},
  SetAttributes[P, Orderless];
  t = Expand[Times @@ K /. {
      X[i_, j_, k_, l_] /; (j == l + 1) ∨ (j == 1 ∧ l == 2 n) :→ q P[i, j] P[k, l] - q^2 P[i, l] P[j, k],
      X[i_, j_, k_, l_] /; (j + 1 == l) ∨ (j == 2 n ∧ l == 1) :→ -q^-2 P[i, j] P[k, l] + q^-1 P[i, l] P[j, k] }];
  Expand[t //. P[i_, j_] P[j_, k_] :→ P[i, k] /. {P[i_, j_]^2 → q + q^-1, P[i_, i_] → q + q^-1}] ];
K1 = PD[X[1, 4, 2, 5] , X[3, 6, 4, 1], X[5, 2, 6, 3]];
K2 = PD[X[1, 6, 2, 7], X[3, 8, 4, 9], X[5, 10, 6, 1], X[7, 2, 8, 3], X[9, 4, 10, 5]];
K3 = PD[X[4, 2, 5, 1], X[8, 4, 9, 3], X[5, 12, 6, 13], X[15, 18, 16, 19], X[9, 16, 10, 17],
  X[17, 10, 18, 11], X[13, 20, 14, 1], X[19, 14, 20, 15], X[11, 6, 12, 7], X[2, 8, 3, 7]];
Jones /@
 {K1,
  K2,
  K3}
```

$$\left\{-\frac{1}{q^9}+\frac{1}{q^5}+\frac{1}{q^3}+\frac{1}{q},\ -\frac{1}{q^{15}}+\frac{1}{q^7}+\frac{1}{q^5}+\frac{1}{q^3},\ -\frac{1}{q^{15}}+\frac{1}{q^7}+\frac{1}{q^5}+\frac{1}{q^3}\right\}$$

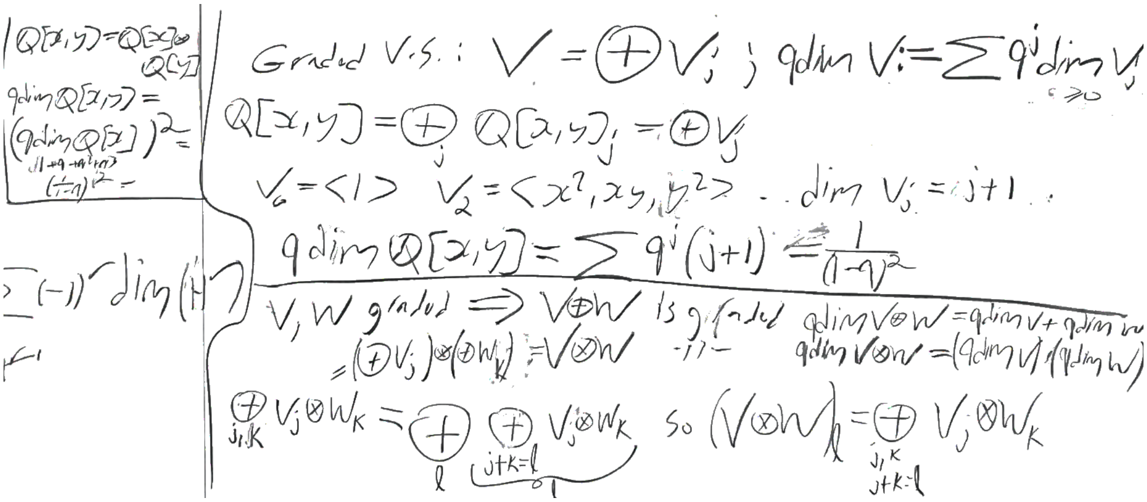
Better Black Boards

```
BBB[m_?MatrixQ] := Module[{k, ker, m1, max, ren},
  k = Ceiling[Norm@Dimensions[m] / 128];
  ker = N[ReplacePart[Table[-1 / (4 k (k + 1)), {2 k + 1}, {2 k + 1}], {k + 1, k + 1} → 1]];
  m1 = ListConvolve[ker, m];
  max = Max[m1];
  ren = If[# < 0.1, 0, (# / max)^(1/2)] &;
  Map[ren, m1, {2}] ];
BBB[img_Image] := Module[{r, g, b},
  Image@Transpose[
   1 - BBB /@ Transpose[ImageData[img], {2, 3, 1}],
   {3, 1, 2} ] ];

BBB[Import["http://drorbn.net/bbs/shots/17-1750-171120-110736.jpg"]]
```



```
BBB[Import["http://drorbn.net/bbs/shots/17-1750-171120-110737.jpg"]]
```



Khovanov Homology

Challenge: Implement Khovanov Homology as it is defined in the handout. Your implementation should fit in one page and should be good enough to show that $Kh(5_1)\neq Kh(10_{132})$.