

Pensieve header: The package “Perm”, day 2.

**Note.** Just one hour today!

**Topics** (in no particular order). Whatever you may suggest; whatever comes to my mind; ~~the Fibonacci numbers; the Catalan numbers; the Jones polynomial; a more efficient Jones algorithm; a riddle on spheres;~~ **Khovanov homology**;  $\Gamma$ -calculus; the Hopf fibration; Hilbert’s 13th problem; **non-commutative Gaussian elimination**; free Lie algebras; the Baker-Campbell-Hausdorff formula; wacky numbers; ~~an order 4 torus;~~ the Schwarz Lantern; knot colourings; the Temperley-Lieb pairing; the dodecahedral link; ~~some experiments;~~ barycentric subdivisions; ~~some Peano curves;~~ braid closures and Vogel’s algorithm; ~~the insolubility of the quintic;~~ phase portraits; **the Mandelbrot set**; shadows of the Cantor aerogel; quilt plots; some image transformations; De Bruijn graphs; the Riemann series theorem; finite type invariants and the Willerton fish; ~~the Towers of Hanoi; Hochschild homology of (some) coalgebras; eevolutions and image improvements;~~ **the 8-5-3 milk jug problem**; ~~a cow problem;~~ **a permutations package.**

## The 8-5-3 Milk Jug Problem

**Problem.** A Milk (chairman  $\rightarrow$  Chair, so milkman  $\rightarrow$  Milk) has three jugs of milk, one carrying 8 liters, one 5 liters, and one 3 liters. The 8 liter jug is full, the other two are empty. Can they measure 4 liters of milk?

**Challenge.** Draw the state graph of this problem (with spilling allowed and also without).

## An NCGE Challenge

Update the NCGE program to contain “backtracking information”. Use it to find how to turn the lower face of a Rubik’s cube by turning all but the lower face of that cube.

## The Mathematica Package Perm

**Challenge.** Re-implement permutations, though using the standard “list of images” notation for permutations: Perm[5,2,3,1,4], etc. Your package should know  $\sigma \circ \tau$ ,  $\sigma^{-1}$ ,  $\sigma[[i]]$ , Pivot[ $\sigma$ ], IdentityPermutation[ $n$ ], it should interact well with Cycles, and its internals should be hidden.

## Some Early Definitions

```
Perm::usage = "Perm[5,2,3,1,4] means the permutation that maps 1→5, 2→2, 3→3, 4→1, 5→4.";
```

```
? Perm
```

```
PermutationQ[ $\sigma$ _Perm] := Sort[List@@ $\sigma$ ] === Range[Length[ $\sigma$ ]]
```

```
PermutationQ /@ {Perm[2, 3, 1], Perm[1, 2, 2]}
```

```
 $\sigma$ _Perm  $\circ$   $\tau$ _Perm /; Length[ $\sigma$ ] == Length[ $\tau$ ] :=  $\sigma$ [[List@@ $\tau$ ]];
```

```
Perm /: ( $\sigma$ _Perm)-1 /; PermutationQ[ $\sigma$ ] := (
   $\tau$  =  $\sigma$ ;
  Do[ $\tau$ [[ $\sigma$ [[i]]]] = i, {i, Length[ $\sigma$ ]}];
   $\tau$ 
)
```

```
Perm[2, 3, 1]-1
```

## Contexts

Every symbol used within a Mathematica session has a “context”.

**? Context**

Context[] gives the current context.  
Context[*symbol*] gives the context in which a symbol appears. >>

**? \$Context**

\$Context is a global variable that gives the current context. >>

```
{Context[], cow = 7, Context[cow], Context[Plus]}
```

```
{Horse`pig = 8, pig, Horse`pig}
```

**? Contexts**

Contexts[] gives a list of all contexts.  
Contexts["*string*"] gives a list of the contexts that match the string. >>

```
Contexts[] // Short
```

**? \$ContextPath**

\$ContextPath is a global variable that gives a list of contexts to search, before \$Context, in trying to find a symbol that has been entered. >>

**\$ContextPath**

```
PrependTo[$ContextPath, "Horse`"];
```

```
pig
```

## The Structure of Packages

**? BeginPackage**

BeginPackage["*context`*"] makes *context`* and System` the only active contexts.  
BeginPackage["*context`*", {"*need*<sub>1</sub>", "*need*<sub>2</sub>", ...}] calls Needs on the *need*<sub>*i*</sub>. >>

**? Begin**

Begin["*context`*"] resets the current context. >>

**? End**

End[] returns the present context, and reverts to the previous one. >>

**? EndPackage**

EndPackage[] restores \$Context and \$ContextPath to their values before the preceding BeginPackage, and prepends the current context to the list \$ContextPath. >>