Pensieve header: October 30: Some further further Hochschild Homology.

Today. Some further further Hochschild homology, (beware of Wolfram the populist!), then a Peano curve, then maybe EIWL 9-12, then, if we're kidding ourselves, Patterns.

Topics (in no particular order). Whatever you may suggest; whatever comes to my mind; ~~the Fibonacci numbers~~; the Catalan numbers; ~~the Jones polynomial~~; ~~a more efficient Jones algorithm~~; ~~a riddle on spheres~~; Khovanov homology; Γ-calculus; the Hopf fibration; Hilbert's 13th problem; non-commutative Gaussian elimination; free Lie algebras; the Baker-Campbell-Hausdorff formula; wacky numbers; an order 4 torus; the Schwarz Lantern; knot colourings; the Temperley-Lieb pairing; the dodecahedral link; sound experiments; barycentric subdivisions; a Peano curve; braid closures and Vogel's algorithm; the insolubility of the quintic; phase portraits; the Mandelbrot set; shadows of the Cantor aerogel; quilt plots; some image transformations; De Bruijn graphs; the Riemann series theorem; finite type invariants and the Willerton fish; ~~the Towers of Hanoi~~; Hochschild homology of (some) coalgebras; convolutions and image improvements.

An Image Manipulation Challenge

The image at http://drorbn.net/bbs/show?shot=17-1750-171016-111042.jpg is pathetic. Can you improve it? Whatever you do, should also work well with all other images at http://drorbn.net/bbs/show.php?prefix=17-1750.

Hochschild Homology of Polynomial Algebras

First see the image at http://drorbn.net/AcademicPensieve/Classes/17-1750-ShamelessMathematica/index.html?im=171023-HomologyBBS.png.

```
d_{n_,k_}[ℰ_] := ℰ /. x_i_ :> Which[i < k, x_i, i == k, x_k + x_{k+1}, i > k, x_{i+1}];
d_n_[ℰ_] := Expand@Sum[(-1)^k d_{n,k}[ℰ], {k, 0, n + 1}];
C_{0,p_} := If[p == 0, {1}, {}];
C_{n_,p_} := C_{n,p} = Union @@ Table[x_n^k C_{n-1,p-k}, {k, 0, p}];
M_{n_,p_} := Transpose[Table[
    da = d_n[a];
    Table[Coefficient[da, b], {b, C_{n+1,p}}],
    {a, C_{n,p}}
   ]];
ρ_{0,_} = 0;
ρ_{n_,p_} := ρ_{n,p} = MatrixRank[M_{n,p}];
β_{n_,p_} := Length[C_{n,p}] - ρ_{n,p} - ρ_{n-1,p};

Table[β_{n,p}, {n, 1, 5}, {p, 1, 5}] // MatrixForm // Timing
```

$$\{0.375,\ \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}\}$$

Hochschild Homology of the Free Associative Algebra

```
w[___, 0, ___] = 0;
w[lft___, α_ * x_i_, rgt___] := α w[lft, x_i, rgt];
w[lft___, a_ + b_, rgt___] := w[lft, a, rgt] + w[lft, b, rgt];

w[3 x_1 + 2 x_2, x_1]
```

```
3 w[x_1, x_1] + 2 w[x_2, x_1]
```

```
Tuples[{-1, 1}, 3]
```

```
{{-1, -1, -1}, {-1, -1, 1}, {-1, 1, -1}, {-1, 1, 1}, {1, -1, -1}, {1, -1, 1}, {1, 1, -1}, {1, 1, 1}}
```

```
tup = Tuples[Table[x_i, {i, 1, 2}], 3]
```

$$\{\{x_1, x_1, x_1\}, \{x_1, x_1, x_2\}, \{x_1, x_2, x_1\}, \{x_1, x_2, x_2\}, \{x_2, x_1, x_1\}, \{x_2, x_1, x_2\}, \{x_2, x_2, x_1\}, \{x_2, x_2, x_2\}\}$$

```
w@tup
```

$$w[\{\{x_1, x_1, x_1\}, \{x_1, x_1, x_2\}, \{x_1, x_2, x_1\}, \{x_1, x_2, x_2\}, \{x_2, x_1, x_1\}, \{x_2, x_1, x_2\}, \{x_2, x_2, x_1\}, \{x_2, x_2, x_2\}\}]$$

```
w @@ tup
```

$$w[\{x_1, x_1, x_1\}, \{x_1, x_1, x_2\}, \{x_1, x_2, x_1\}, \{x_1, x_2, x_2\}, \{x_2, x_1, x_1\}, \{x_2, x_1, x_2\}, \{x_2, x_2, x_1\}, \{x_2, x_2, x_2\}]$$

```
w @@@ tup
```

$$\{w[x_1, x_1, x_1], w[x_1, x_1, x_2], w[x_1, x_2, x_1], w[x_1, x_2, x_2], w[x_2, x_1, x_1], w[x_2, x_1, x_2], w[x_2, x_2, x_1], w[x_2, x_2, x_2]\}$$

```
NC_n_,p_ := w @@@ Tuples[Table[x_i, {i, 1, n}], p]
```

```
NC_3,2
```

$$\{w[x_1, x_1], w[x_1, x_2], w[x_1, x_3], w[x_2, x_1], w[x_2, x_2], w[x_2, x_3], w[x_3, x_1], w[x_3, x_2], w[x_3, x_3]\}$$

```
NC_3,2 // d_3
```

$$\{-w[x_1, x_2] - w[x_2, x_1], w[x_1, x_2], -w[x_1, x_4], w[x_2, x_1], w[x_2, x_2] + w[x_2, x_3] + w[x_3, x_2] + w[x_3, x_3], w[x_3, x_4], -w[x_4, x_1], w[x_4, x_3], -w[x_3, x_4] - w[x_4, x_3]\}$$

```
NC_3,2 // d_3 // d_4
```

$$\{0, 0, 0, 0, 0, 0, 0, 0, 0\}$$

```
NM_n_,p_ := Transpose[Table[
    da = d_n[a];
    Table[Coefficient[da, b], {b, NC_n+1,p}],
    {a, NC_n,p}
   ]];
Nρ_0,_ = 0;
Nρ_n_,p_ := Nρ_n,p = MatrixRank[NM_n,p];
Nβ_n_,p_ := Length[NC_n,p] - Nρ_n,p - Nρ_n-1,p;
```

```
Table[Nβ_n,p, {n, 1, 4}, {p, 1, 4}] // MatrixForm // Timing
```

$$\{0., \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}\}$$