

Pensieve header: Sep 29: The Catalan numbers.

Topics (in no particular order). Whatever you may suggest; whatever comes to my mind; ~~the Fibonacci numbers~~; ~~the Catalan numbers~~; ~~the Jones polynomial~~; a more efficient Jones algorithm; a riddle on spheres; Khovanov homology; Γ -calculus; the Hopf fibration; Hilbert's 13th problem; non-commutative Gaussian elimination; free Lie algebras; the Baker-Campbell-Hausdorff formula; wacky numbers; an order 4 torus; the Schwarz Lantern; knot colourings; the Temperley-Lieb pairing; the dodecahedral link; sound experiments; barycentric subdivisions; a Peano curve; braid closures and Vogel's algorithm; the insolubility of the quintic; phase portraits; the Mandelbrot set; shadows of the Cantor Aerogel; quilt plots; some image transformations; De Bruijn graphs; the Riemann series theorem; finite type invariants and the Willerton fish.

Killing Time

```
IntegerDigits[371]
```

```
{3, 7, 1}
```

```
IntegerDigits[371]^3
```

```
{27, 343, 1}
```

```
Total[IntegerDigits[371]^3]
```

```
371
```

```
crit[n_] := (n == Total[IntegerDigits[n]^3])
```

```
Select[Range[100, 999], crit]
```

```
{153, 370, 371, 407}
```

```
Select[Range[100, 999], crit] // Length
```

```
4
```

The Catalan Numbers

Project Idea. Make an easily extendible, n -dependent “Catalan objects poster”, as explained on the blackboard.

```
Clear[ts]
```

```
ts[n_Integer] := ts[Range[0, n + 1]]
```

```
ts[4]
```

```
ts[{0, 1, 2, 3, 4, 5}]
```

```
leon[_] := 7
```

```
leon[dror]
```

```
7
```

```
dror[n_] := n2
```

```
dror[7]
```

```
49
```

```
dror[Cos[x]]
```

```
Cos[x]2
```

```
hang[n_Integer] := n2;
```

```
hang[n_Symbol] := n3
```

```
hang[4]
```

```
16
```

```
hang[dror]
```

```
dror3
```

```
Union[{1, 2, 3}, {4, 5, 6}]
```

```
{1, 2, 3, 4, 5, 6}
```

```
vs = {a, b, c, d, e, f}; k = 4;
```

```
Table[vs[[j]], {j, k, Length[vs]}]
```

```
{d, e, f}
```

? Prepend

```
Prepend[expr, elem] gives expr with elem prepended.  
Prepend[elem] represents an  
operator form of Prepend that can be applied to an expression. >
```

```
Prepend[Table[vs[[j]], {j, k, Length[vs]}], vs[[1]]]
```

```
{a, d, e, f}
```

```
Prepend[vs[[k ;;]], vs[[1]]]
```

```
{a, d, e, f}
```

```
vs[[2 ;; k]]
```

```
{b, c, d}
```

```
l = {a, b, c}; r = {x, y, z};
```

```

Flatten[Table[
  Table[
    f[l[[i]], r[[j]],
    {j, 1, Length[r]}
  ],
  {i, 1, Length[l]}
]]
{f[a, x], f[a, y], f[a, z], f[b, x], f[b, y], f[b, z], f[c, x], f[c, y], f[c, z]}

```

```

Flatten[Table[
  f[l[[i]], r[[j]],
  {i, 1, Length[l]}, {j, 1, Length[r]}
]]
{f[a, x], f[a, y], f[a, z], f[b, x], f[b, y], f[b, z], f[c, x], f[c, y], f[c, z]}

```

```

Flatten[Table[
  f[l[[i]], r[[j]],
  {i, Length[l]}, {j, Length[r]}
]]
{f[a, x], f[a, y], f[a, z], f[b, x], f[b, y], f[b, z], f[c, x], f[c, y], f[c, z]}

```

```

Flatten[Table[f[t1, t2], {t1, l}, {t2, r}]]
{f[a, x], f[a, y], f[a, z], f[b, x], f[b, y], f[b, z], f[c, x], f[c, y], f[c, z]}

```

```

ts[[_ , _]] = {ds[]};
ts[vs_List] := Module[{l, r, k, t1, t2, tds},
  Union@@ Table[
    l = ts[Prepend[vs[[k ;;]], vs[[1]]]];
    r = ts[vs[[2 ;; k]]];
    Flatten[Table[
      tds = Join[t1, t2];
      If[k > 3, AppendTo[tds, d[vs[[2]], vs[[k]]]];
      If[k < Length[vs], AppendTo[tds, d[vs[[1]], vs[[k]]]];
      tds,
      {t1, l}, {t2, r}
    ]],
    {k, 3, Length[vs]}
  ]
]

```

```
ts[{} , 1]
```

```
{ds[]}
```

```
ts[{} , 1, 2]
```

```
{ds[]}
```

```
ts[{} , 1, 2, 3]
```

```
{ds[d[0, 2]], ds[d[1, 3]]}
```

ts[2]

```
{ds[d[0, 2]], ds[d[1, 3]]}
```

ts[3]

```
{ds[d[0, 3], d[0, 2]], ds[d[1, 3], d[0, 3]],
 ds[d[1, 3], d[1, 4]], ds[d[2, 4], d[0, 2]], ds[d[2, 4], d[1, 4]]}
```

ts[4]

```
{ds[d[0, 4], d[0, 3], d[0, 2]], ds[d[0, 4], d[1, 3], d[0, 3]],
 ds[d[1, 3], d[1, 4], d[0, 4]], ds[d[1, 4], d[1, 3], d[1, 5]], ds[d[2, 4], d[0, 4], d[0, 2]],
 ds[d[2, 4], d[1, 4], d[0, 4]], ds[d[2, 4], d[1, 4], d[1, 5]], ds[d[2, 4], d[2, 5], d[0, 2]],
 ds[d[2, 4], d[2, 5], d[1, 5]], ds[d[3, 5], d[0, 3], d[0, 2]], ds[d[3, 5], d[1, 3], d[0, 3]],
 ds[d[3, 5], d[1, 3], d[1, 5]], ds[d[3, 5], d[2, 5], d[0, 2]], ds[d[3, 5], d[2, 5], d[1, 5]]}
```

ts[4] // Length

14

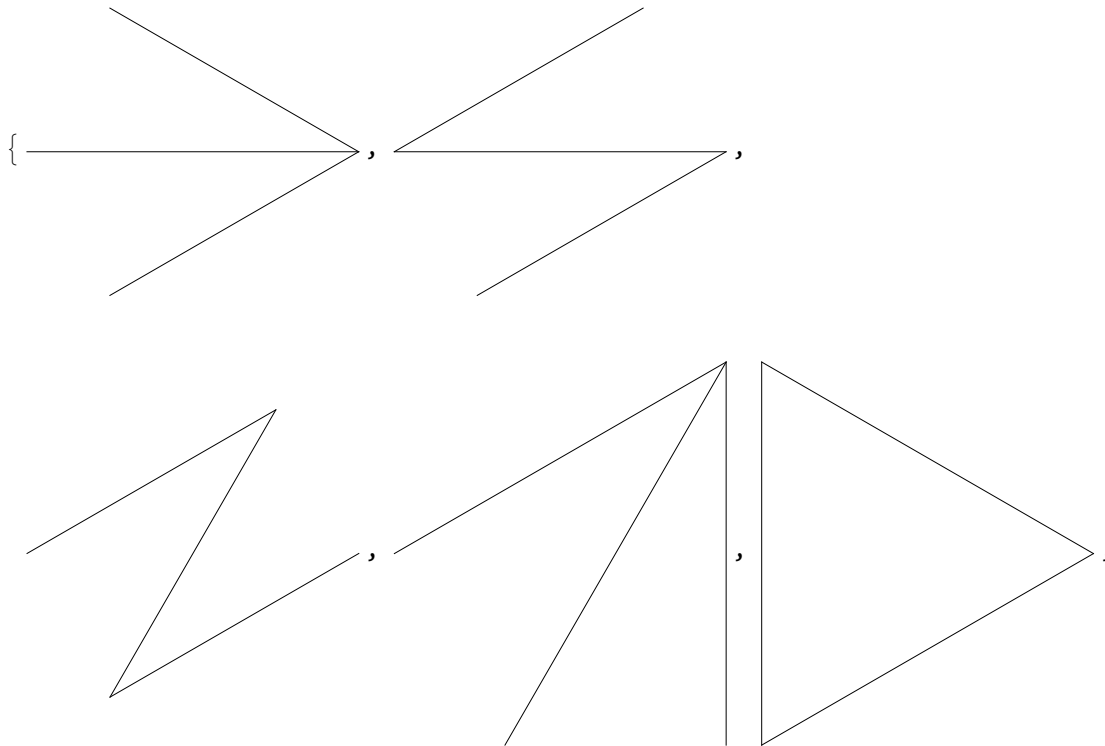
ts[5] // Length

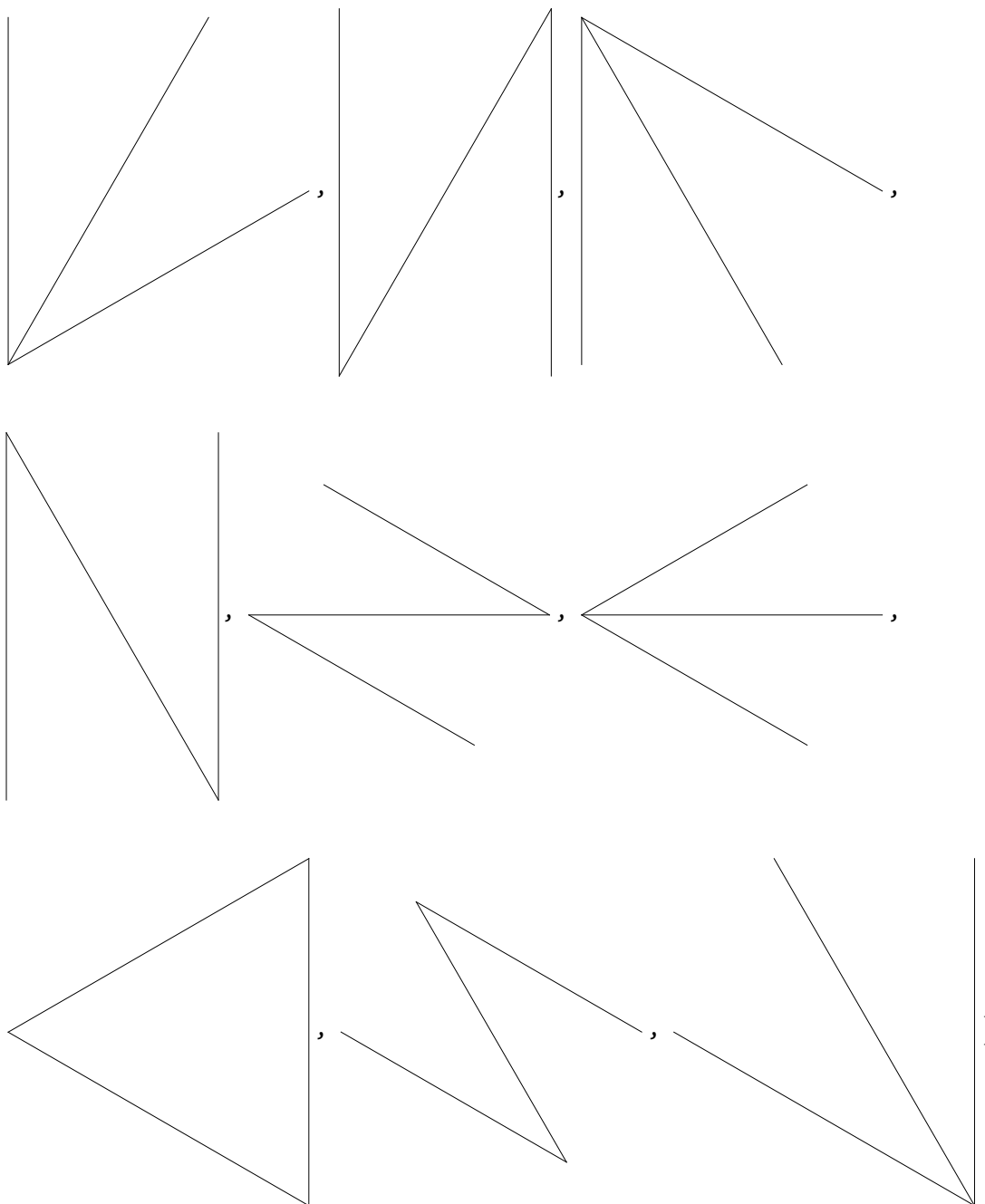
42

ts[6] // Length

132

```
ts[4] /. ds[ls___] -> Graphics[{ls}] /. d[i_, j_] -> Line[{i, j}] /.
 j_Integer -> {Cos[2πj/6], Sin[2πj/6]}
```





Further topics

Class photo; EIWL 5-8, a riddle on spheres, Etienne's project.