

Pensieve header: A Drinfel'd associator Φ to degree 8.

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Get["http://drorbn.net/AcademicPensieve/Projects/WK04/FreeLie.m"];
qs[2, 1] = qs[3, 1] = qs[3, 2] = 0; qs[3, 1, 2] = 1/24; q = DKS[3, qs];
SeriesSolve[q, q^σ[3,2,1] ≡ -q ∧ q ** q^σ[1,23,4] ** q^σ[2,3,4] ≡ q^σ[12,3,4] ** q^σ[1,2,34]];
q@{6}
```

FreeLie` implements / extends

{*, +, **, \$SeriesShowDegree, ⟨⟩, ∫, ≡, ad, Ad, adSeries, AllCyclicWords, AllLyndonWords, AllWords, Arbitrator, ASeries, AW, b, BCH, BooleanSequence, BracketForm, BS, CC, Crop, cw, CW, CWS, CWSeries, D, Deg, DegreeScale, DerivationSeries, div, DK, DKS, DKSeries, EulerE, Exp, Inverse, j, J, JA, LieDerivation, LieMorphism, LieSeries, LS, LW, LyndonFactorization, Morphism, New, RandomCWSeries, Randomizer, RandomLieSeries, RC, SeriesSolve, Support, t, tb, TopBracketForm, tr, UndeterminedCoefficients, αMap, Γ, ℓ, Δ, σ, ħ, ↦, ↵}.

FreeLie` is in the public domain. Dror Bar-Natan is

committed to support it within reason until July 15, 2022. This is version 150814.

SeriesSolve: In degree 3 arbitrarily setting {Φs[3, 1, 1, 2] → 0}.



SeriesSolve: In degree 5 arbitrarily setting {Φs[3, 1, 1, 1, 1, 2] → 0}.



$$\text{DKS}\left[0, \frac{1}{24} \overbrace{t_{13} t_{23}}^{\overbrace{t_{13} t_{23}}}, 0, -\frac{7}{5760} \overbrace{t_{13} t_{23} t_{23} t_{23}}^{\overbrace{t_{13} t_{23} t_{23} t_{23}}}, +\frac{7}{5760} \overbrace{t_{13} t_{13} t_{23} t_{23}}^{\overbrace{t_{13} t_{13} t_{23} t_{23}}}, -\frac{t_{13} t_{13} t_{13} t_{23}}{1440}, 0, \frac{31}{967680} \overbrace{t_{13} t_{23} t_{23} t_{23} t_{23} t_{23}}^{\overbrace{t_{13} t_{23} t_{23} t_{23} t_{23} t_{23}}}, \right.$$

$$\frac{157}{1935360} \overbrace{t_{13} t_{13} t_{23} t_{23} t_{13} t_{23}}^{\overbrace{t_{13} t_{13} t_{23} t_{23} t_{13} t_{23}}}, -\frac{31}{387072} \overbrace{t_{13} t_{23} t_{13} t_{23} t_{23} t_{23}}^{\overbrace{t_{13} t_{23} t_{13} t_{23} t_{23} t_{23}}}, -\frac{31}{483840} \overbrace{t_{13} t_{13} t_{23} t_{23} t_{23} t_{23}}^{\overbrace{t_{13} t_{13} t_{23} t_{23} t_{23} t_{23}}}, +\frac{11}{290304} \overbrace{t_{13} t_{13} t_{13} t_{23} t_{13} t_{23}}^{\overbrace{t_{13} t_{13} t_{13} t_{23} t_{13} t_{23}}},$$

$$\left. \frac{31}{725760} \overbrace{t_{13} t_{13} t_{23} t_{13} t_{23} t_{23}}^{\overbrace{t_{13} t_{13} t_{23} t_{13} t_{23} t_{23}}}, +\frac{83}{967680} \overbrace{t_{13} t_{13} t_{13} t_{23} t_{23} t_{23}}^{\overbrace{t_{13} t_{13} t_{13} t_{23} t_{23} t_{23}}}, -\frac{13}{241920} \overbrace{t_{13} t_{13} t_{13} t_{13} t_{23} t_{23}}^{\overbrace{t_{13} t_{13} t_{13} t_{13} t_{23} t_{23}}}, +\frac{t_{13} t_{13} t_{13} t_{13} t_{13} t_{23}}{60480}, \dots \right]$$

Timing [Ⓜ@{8}]

SeriesSolve: In degree 7 arbitrarily setting {Φs[3, 1, 1, 1, 1, 1, 2] → 0}.



SeriesSolve: In degree 8 arbitrarily setting {Φs[3, 1, 1, 1, 1, 1, 2, 1, 2] → 0}.



$$\left\{ 137.797, \text{DKS} \left[0, \frac{1}{24} t_{13} t_{23}, 0, -\frac{7 t_{13} t_{23} t_{23} t_{23}}{5760} + \frac{7 t_{13} t_{13} t_{23} t_{23}}{5760} - \frac{t_{13} t_{13} t_{13} t_{23}}{1440}, 0, \right. \right.$$

$$\frac{31 t_{13} t_{23} t_{23} t_{23} t_{23}}{967680} - \frac{157 t_{13} t_{13} t_{23} t_{23} t_{13} t_{23}}{1935360} - \frac{31 t_{13} t_{23} t_{13} t_{23} t_{23} t_{23}}{387072} - \frac{31 t_{13} t_{13} t_{23} t_{23} t_{23} t_{23}}{483840} +$$

$$\frac{11 t_{13} t_{13} t_{13} t_{23} t_{13} t_{23}}{290304} + \frac{31 t_{13} t_{13} t_{23} t_{13} t_{23} t_{23}}{725760} + \frac{83 t_{13} t_{13} t_{13} t_{23} t_{23} t_{23}}{967680} - \frac{13 t_{13} t_{13} t_{13} t_{13} t_{23} t_{23}}{241920} +$$

$$\frac{t_{13} t_{13} t_{13} t_{13} t_{13} t_{23}}{60480}, 0, -\frac{127 t_{13} t_{23} t_{23} t_{23} t_{23} t_{23} t_{23}}{154828800} + \frac{503 t_{13} t_{13} t_{23} t_{23} t_{13} t_{23} t_{13} t_{23}}{69672960} +$$

$$\frac{71 t_{13} t_{23} t_{13} t_{23} t_{23} t_{13} t_{23} t_{23}}{9289728} + \frac{127 t_{13} t_{23} t_{23} t_{13} t_{23} t_{23} t_{23} t_{23}}{22118400} + \frac{3613 t_{13} t_{13} t_{23} t_{23} t_{23} t_{23} t_{13} t_{23}}{464486400} +$$

$$\frac{7 t_{13} t_{13} t_{23} t_{23} t_{23} t_{13} t_{23} t_{23}}{737280} + \frac{107 t_{13} t_{13} t_{23} t_{13} t_{23} t_{23} t_{13} t_{23}}{29030400} - \frac{251 t_{13} t_{13} t_{13} t_{23} t_{23} t_{13} t_{13} t_{23}}{116121600} -$$

$$\frac{881 t_{13} t_{13} t_{23} t_{13} t_{13} t_{23} t_{13} t_{23}}{174182400} - \frac{209 t_{13} t_{13} t_{23} t_{13} t_{13} t_{23} t_{23} t_{23}}{51609600} + \frac{127 t_{13} t_{23} t_{13} t_{23} t_{23} t_{23} t_{23} t_{23}}{22118400} -$$

$$\frac{199 t_{13} t_{23} t_{13} t_{23} t_{13} t_{23} t_{23} t_{23}}{21772800} + \frac{127 t_{13} t_{13} t_{23} t_{23} t_{23} t_{23} t_{23} t_{23}}{51609600} - \frac{367 t_{13} t_{13} t_{13} t_{23} t_{13} t_{23} t_{13} t_{23}}{69672960} -$$

$$\frac{6439 t_{13} t_{13} t_{23} t_{23} t_{13} t_{23} t_{23} t_{23}}{696729600} - \frac{25577 t_{13} t_{13} t_{13} t_{23} t_{23} t_{23} t_{13} t_{23}}{1393459200} - \frac{55 t_{13} t_{13} t_{13} t_{23} t_{23} t_{13} t_{23} t_{23}}{3981312} +$$

$$\frac{163 t_{13} t_{13} t_{13} t_{13} t_{23} t_{13} t_{13} t_{23}}{58060800} + \frac{67 t_{13} t_{13} t_{13} t_{23} t_{13} t_{13} t_{23} t_{23}}{11612160} - \frac{2003 t_{13} t_{13} t_{23} t_{13} t_{23} t_{23} t_{23} t_{23}}{139345920} +$$

$$\frac{673 t_{13} t_{13} t_{23} t_{13} t_{23} t_{13} t_{23} t_{23}}{43545600} - \frac{2399 t_{13} t_{13} t_{13} t_{23} t_{23} t_{23} t_{23} t_{23}}{464486400} + \frac{2693 t_{13} t_{13} t_{13} t_{13} t_{23} t_{23} t_{13} t_{23}}{348364800} +$$

$$\frac{15859 t_{13} t_{13} t_{13} t_{23} t_{13} t_{23} t_{23} t_{23}}{1393459200} + \frac{2893 t_{13} t_{13} t_{13} t_{13} t_{23} t_{23} t_{23} t_{23}}{464486400} - \frac{1007 t_{13} t_{13} t_{13} t_{13} t_{23} t_{13} t_{23} t_{23}}{348364800} -$$

$$\frac{271 t_{13} t_{13} t_{13} t_{13} t_{13} t_{23} t_{23} t_{23}}{58060800} + \frac{19 t_{13} t_{13} t_{13} t_{13} t_{13} t_{13} t_{23} t_{23}}{9676800} - \frac{t_{13} t_{13} t_{13} t_{13} t_{13} t_{13} t_{13} t_{23}}{2419200}, \dots \left. \right\}$$

(A computation to degree 10 is available at <http://drorbn.net/AcademicPensieve/Projects/WKO4/nb/Phi.pdf>. It takes about 8 hours and prints on about 6 pages).