

Pensieve header: Computing the Jones polynomial, rough procedure.

First, Class Introduction as in <http://drorbn.net/AcademicPensieve/Classes/17-1350-AKT/About.html>.

The Jones Polynomial of the Trefoil Knot

Jones: $\nearrow \mapsto q$ $(-q^2 \searrow)$ $\bigcirc \mapsto q + q^{-1}$

$\nwarrow \mapsto -q^{-2} \searrow + q^{-1}$

Once[<< KnotTheory`]

K = Knot[3, 1]

PD[K]

t1 = PD[K] /. x_X => If[PositiveQ[x], Xp@@x, Xm@@x]

t2 = Times@@t1

t3 = t2 /. {
 Xp[i_, j_, k_, l_] => q * P[i, j] P[k, l] - q^2 * P[i, l] P[j, k],
 Xm[i_, j_, k_, l_] => -q^(-2) * P[i, j] P[k, l] + 1/q * P[i, l] P[j, k]
 }

t4 = Expand[t3]

SetAttributes[P, Orderless]

t5 = t4 //. P[a_, b_] P[b_, c_] => P[a, c]

t6 = t5 /. {
 P[i_, i_] => (q + 1/q),
 P[i_, j_]^2 => (q + 1/q)
 }

Simplify[t6]

Jones[Knot[3, 1]][q]

Simplify[$\frac{t6}{q + q^{-1}}$ /. q -> \sqrt{q}]

A Jones Polynomial Program

```
J[K_] := Module[{t1, t2, t3, t4, t5, t6, P},
  SetAttributes[P, Orderless];
  t1 = PD[K] /. x_X => If[PositiveQ[x], Xp@@x, Xm@@x];
  t2 = Times@@t1;
  t3 = t2 /. {
    Xp[i_, j_, k_, L_] => q * P[i, j] P[k, L] - q^2 * P[i, L] P[j, k],
    Xm[i_, j_, k_, L_] => -q^(-2) * P[i, j] P[k, L] + 1/q * P[i, L] P[j, k]
  };
  t4 = Expand[t3];
  t5 = t4 //. P[a_, b_] P[b_, c_] => P[a, c];
  t6 = t5 /. {
    P[i_, i_] => (q + 1/q),
    P[i_, j_]^2 => (q + 1/q)
  };
  Simplify[ $\frac{t6}{q + q^{-1}}$  /. q ->  $\sqrt{q}$ ]
]

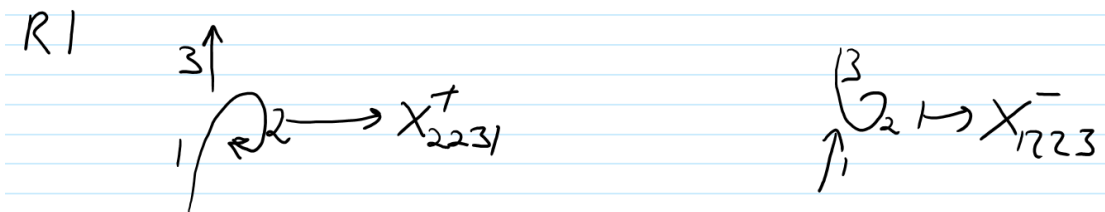
J[Knot[8, 17]]

Jones[Knot[8, 17]][q]

(K -> Expand[J[K]] == Jones[K][q]) /@ AllKnots[{3, 10}]
```

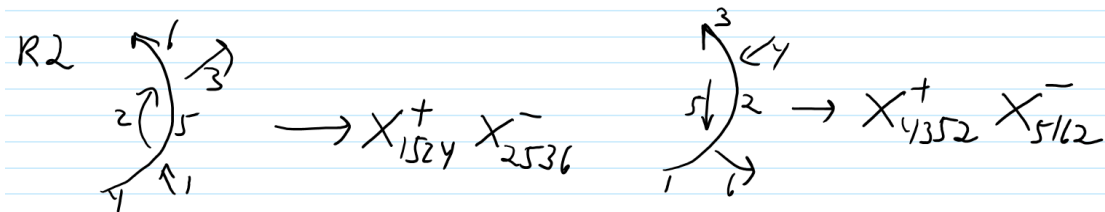
Testing the Reidemeister Moves

```
SetAttributes[P, Orderless];
TJ[t2_] := Module[{t3, t4, t5, t6},
  t3 = t2 /. {
    Xp[i_, j_, k_, L_] => q * P[i, j] P[k, L] - q^2 * P[i, L] P[j, k],
    Xm[i_, j_, k_, L_] => -q^(-2) * P[i, j] P[k, L] + 1/q * P[i, L] P[j, k]
  };
  t4 = Expand[t3];
  t5 = t4 //. P[a_, b_] P[b_, c_] => P[a, c];
  t6 = t5 /. {
    P[i_, i_] => (q + 1/q),
    P[i_, j_]^2 => (q + 1/q)
  };
  Simplify[t6]
]
```

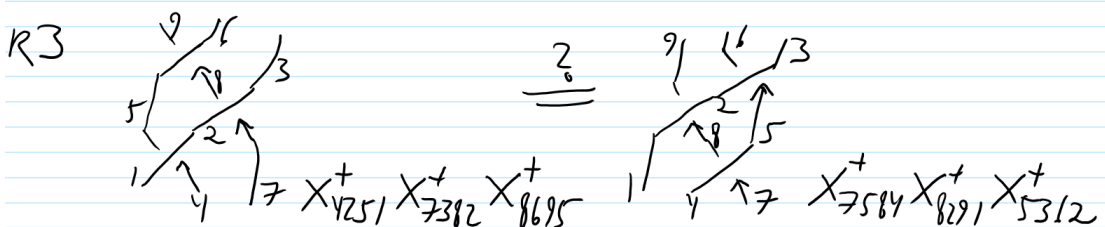


Xp[2, 2, 3, 1] // TJ

Xm[1, 2, 2, 3] // TJ



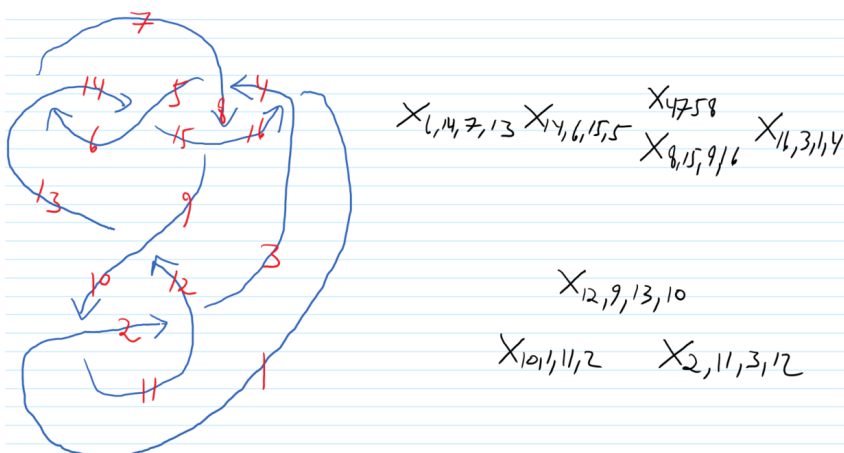
TJ /@ {Xp[1, 5, 2, 4] Xm[2, 5, 3, 6], Xp[4, 3, 5, 2] Xm[5, 1, 6, 2]}



TJ /@ {Xp[4, 2, 5, 1] Xp[7, 3, 8, 2] Xp[8, 6, 9, 5], Xp[7, 5, 8, 4] Xp[8, 2, 9, 1] Xp[5, 3, 6, 2]}

%[[1] == %[[2]

Analyzing a knot suggested by David Vincent



K1 = PD[X[6, 14, 7, 13], X[14, 6, 15, 5], X[4, 7, 5, 8], X[8, 15, 9, 16], X[16, 3, 1, 4], X[12, 9, 13, 10], X[10, 1, 11, 2], X[2, 11, 3, 12]];

```
J1 = J[K1]
```

```
Select[AllKnots[{3, 8}], J[#] == J1 &]
```