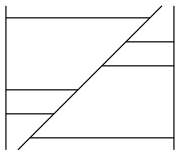
Associators and Multiple Zeta Values



$$\longrightarrow \quad \int_{0\le t_1\le\ldots\le t_6\le 1} \frac{d(1-t_1)}{1-t_1}\frac{dt_2}{t_2}\frac{dt_3}{t_3}\frac{d(1-t_4)}{1-t_4}\frac{d(1-t_5)}{1-t_5}\frac{dt_6}{t_6}$$

$$= -\int_0^1 \frac{dt_6}{t_6}\int_0^{t_6} \frac{dt_5}{1-t_5}\int_0^{t_5} \frac{dt_4}{1-t_4}\int_0^{t_4} \frac{dt_3}{t_3}\int_0^{t_3} \frac{dt_2}{t_2}\int_0^{t_2} \frac{dt_1}{1-t_1}$$

$$\sum_{k_1>0} t_1^{k_1-1}$$

$$\sum_{k_1>0} \frac{t_2^{k_1}}{k_1}$$

$$\sum_{k_1>0} \frac{t_3^{k_1}}{k_1^2}$$

$$\sum_{k_1>0} \frac{t_4^{k_1}}{k_1^3}$$

$$\sum_{k_1,k_2>0} \frac{t_5^{k_1+k_2}}{k_1^3(k_1+k_2)}$$

$$\sum_{k_1,k_2,k_3>0} \frac{t_6^{k_1+k_2+k_3}}{k_1^3(k_1+k_2)(k_1+k_2+k_3)}$$

$$= -\sum_{k_1,k_2,k_3>0} \frac{1^{k_1+k_2+k_3}}{k_1^3(k_1+k_2)(k_1+k_2+k_3)^2}$$

$$= -\sum_{0<n_1<n_2<n_3} \frac{1}{n_1^3 n_2 n_3^2} =: -\zeta(3,1,2).$$

Tuesday's class: