1617-257 Wed Sep 28, Hour 8: Compactness, 2.

September 18, 2016 3:11 PM

class photo on Friday!
Read Along: sect 4
Today's agenda: compactness in $\mathbb{R}^n$.

Riddle Along: Can you find uncountably many subsets of $\mathbb{N}$ s.t. for any two of them A & B, $(A\subset B)\vee(B\subset A)$?

Def "$A\subset X$ is compact" means

$$\left(A=\bigcup_{\alpha\in I}V_\alpha,\ V_\alpha \text{ open in } A\right)\Rightarrow\left(\exists F\subset I \text{ finite, s.t. } A=\bigcup_{\alpha\in F}V_\alpha\right)$$

$$\Leftrightarrow$$

$$\left(A\subset\bigcup_{\alpha\in I}U_\alpha,\ U_\alpha \text{ open in } X\right)\Rightarrow\left(\exists F\subset I \text{ finite, s.t. } A\subset\bigcup_{\alpha\in F}U_\alpha\right)$$

No compact: $\mathbb{R}$, $(0,1)$

compact: Finite, $[0,1]$, $[a,b]$

A cont. functn on a compact set is bndd.

pre-write

Thm $X\subset\mathbb{R}^n$ is compact iff it is closed and bndd.

Pf $\Rightarrow$

$\Leftarrow$ **Lemma** X,Y compact $\Rightarrow$ $X\times Y$ compact.

"Tychonoff's thm"

(Some discussion of $X\times Y$) Notes: 1. Can use sup or Euclidean 2. Every open set is a union of "open squares".

Lemma² If U_α is an open cover of $X\times Y$, then for every x, there is an open $Z_x\ni x$, s.t. $Z_x\times Y$ is covered by finitely many of the U_α's.

Lemma² done, lemma² to lemma not yet.

So by induction, $\prod[a_i,b_i]$ is compact.

Lemma A closed subset of a compact set is compact

$\square\square$

Thm A cont. image of a compact set is compact

Thm The maximal value thm. Also, "extremal value thm".

Thm The ϵ-nbd theorem: If C is compact and $U\supset C$ is open, then there is some $\epsilon>0$ s.t. $U(C,\epsilon):=\{x: d(C,x)<\epsilon\}\subset U$ where $d(C,x):=\inf_{y\in C}d(x,y)$.

Thm Uniform continuity.