1617-257 Mon Sep 26, Hour 7: Compactness.

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All who asked for a wiki account got it?

Today's agenda: "Compactness".

Riddle Along: In how many ways can you place 4 points in the Euclidean plane so that there would be exactly two different distances between them? pre-write

Def A space X is called "compact", if whenever you cover X with open sets, finitely many of these already cover X:

$$\left(X \subset \bigcup_{\alpha\in I} U_\alpha,\ U_\alpha \text{ open}\right) \Longrightarrow \left(\exists F\subset I \text{ finite, s.t. } X \subset \bigcup_{\alpha\in F} U_\alpha\right)$$

Compactness for subsets.

Examples 1. Finite is compact.

2. $\mathbb{R}$ is not compact.

3. $(0,1)$ is not compact.

Thm $I=[0,1]$ is compact.

Pf Suppose U_α, $\alpha\in I$ is an open cover of I.

Let $B=\{y\in I : [0,y] \text{ can be covered by finitely many of the } U_\alpha\text{'s}\}$

Then $0\in B$ and B is bounded. Let $b=\sup B$.

1. $b\in B$ 0. $b>0$

2. $b=1$

$\square$

Thm A compact subset of $\mathbb{R}^n$ is closed and bounded. not done

done line

Thm A cont. image of a compact set is compact is compact

Thm The maximal value thm.

If time, the \epsilon-neighborhood theorem.