# 1617-257 Mon Oct 17, Hour 15: the Chain Rule

October 14, 2016 6:25 AM

Riddle Along (Kodiak Jackson): An ant walks at 1cm/s on a 1m super-rubber band that stretches at 1m/s. Will it get to the other end? Why not? When?

Read Along Sec 7, 8

Reminder: $f(a+h) = f(a) + Df(a)\cdot h + \gamma(h)$, $\gamma \in o(h)$. on board

**Thm (The chain rule):** If $a \in \mathbb{R}^n \xrightarrow{f} \mathbb{R}^m \xrightarrow{g} \mathbb{R}^p$, $f$ diffable at $x$, $g$ diffable at $f(a)$,

then $D(g\circ f)(a) = Dg(f(a))\,Df(a)$

**Example** $\mathbb{R}_t \xrightarrow{(t,t)} \mathbb{R}^2_{x,y} \xrightarrow{x^y} \mathbb{R}$

**Pf of Thm** $f(a+h) = f(a) + F\cdot h + \phi(h)$ $\quad \phi \in o(h)$

$b = f(a)$ $\quad g(b+k) = g(b) + Gk + \gamma(k)$ $\quad \gamma \in o(k)$

$$(g\circ f)(a+h) = g(f(a+h)) = g(\underbrace{f(a)}_{b} + \underbrace{Fh + \phi(h)}_{k}) = g(b) + Gk + \gamma(k)$$

$$= g(f(a)) + GFh + \underbrace{G\cdot\phi(h) + \gamma(Fh + \phi(h))}_{\lambda(h)} = g(f(a)) + GFh + \lambda(h)$$

Claim $\lambda(h) \in o(h)$ } gap here

**Corollary 1.** A composition of $C^r$ functions is $C^r$. [Note domains!] done in line.

**Corollary 2.** [Mvt in $\mathbb{R}^n$, skip]

**Corollary 3.** If $f, g : \mathbb{R}^n \to \mathbb{R}^n$, $f$ diffable at $a$, $g$ diffable at $b = f(a)$, and $g(f(x)) = x$ near $a$, then $Dg(b) = [Df(a)]^{-1}$