

class photo at 14:55

HW1 due, HW2 on web by midnight

Read Along: Sect 4

Today's agenda: compactness, connectedness.

Riddle Along: In a random 13-element subset of $\{1, \dots, 52\}$,

what is the average value of the smallest element?

Example: 2 in 3: $\{1,2\}, \{1,3\}, \{2,3\} \rightarrow \frac{1+1+2}{3} = \frac{4}{3}$

Thm $X \subset \mathbb{R}^n$ is compact iff it is closed and bdd.

pf \Rightarrow done.

Lemma: If X & Y are compact, then so is $X \times Y$.

\Leftarrow Lemma² If $W_k = U_k \times V_k$ is an open cover of $X \times Y$

then for every x , there is an open $Z_x \ni x$, s.t. $Z_x \times Y$ is covered by finitely many of the W_k 's.

pre-write

Lemma² \Rightarrow Lemma

Lemma \Rightarrow $[a_i, b_i]$ is compact.

Lemma A closed subset of a compact set is compact
□□

Thm A cont. image of a compact set is compact

Thm the maximal value thm. Also, "extremal value thm".

Thm The ϵ -nsd theorem: If C is compact and $U \supset C$ is open, then there is some $\epsilon > 0$ s.t. $U(C, \epsilon) := \{x : d(C, x) < \epsilon\} \subset U$ where $d(C, x) := \inf_{y \in C} d(x, y)$.

skipped

Thm Uniform continuity.

Def X is "connected" if the only clopen sets in X are \emptyset & X . Alternatively, if whenever $X = A \cup B$ w/ $A \cap B = \emptyset$ and A & B are open, then at least one of A & B is empty.

skipped.

Thm A subset of \mathbb{R} is connected iff it is empty, an interval, a ray, or the whole thing.

Thm A cont. image of a connected set is connected.

Thm The intermediate-value thm: If X is connected, $f: X \rightarrow \mathbb{R}$ is cont., $f(a) < 0$ $f(b) > 0$, then $\exists x \in X$ s.t. $f(x) = 0$.