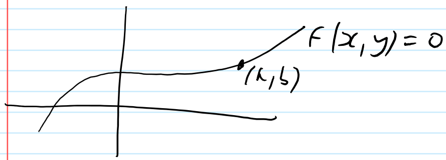
1617-257 Fri Oct 28, Hour 20: The Implicit Function Theorem, Integration.

October 14, 2016 6:25 AM

HW4 returned, HW5 due. Post solutions!
Read Along: Secs 9.
TT: Tue Nov 1 5PM-7PM @ BI 131. Extra OH: Jeff Mon 4-7 Huron 215 10th floor, Dror Tue 11-2 BA 6178,
Agenda: as above.



Find $y=g(x)$ s.t. $f(x,g(x))=0$.

__Thm__ Given a C^r $f:\mathbb{R}^n_{x_1\dots x_n}\times\mathbb{R}^k_{y_1\dots y_k}\longrightarrow\mathbb{R}^k$ and $(a,b)\in\mathbb{R}^n\times\mathbb{R}^k$ st. $f(a,b)=0$ &

there exists a unique C^r $g:\left\{\begin{matrix}\text{nbd } U\\ \text{of } a\end{matrix}\right\}\to\left\{\begin{matrix}\text{nbd of}\\ b\end{matrix}\right\}$ s.t. $g(a)=b$ & $\forall z\in U$ $f(z,g(z))=0$.

Furthermore, $Dg=$

on board

__pf__ $f(z,y)=0 \iff \begin{cases}x=z\\ f(x,y)=0\end{cases}$ so with $H(x,y):=\begin{pmatrix}x\\ f(x,y)\end{pmatrix}$

this is $H\begin{pmatrix}x\\y\end{pmatrix}=\begin{pmatrix}z\\0\end{pmatrix}$ where $H\begin{pmatrix}a\\b\end{pmatrix}=\begin{pmatrix}a\\0\end{pmatrix}$. Assuming $DH\begin{pmatrix}a\\b\end{pmatrix}$ is non-singular, H^{-1} exists near $\begin{pmatrix}a\\0\end{pmatrix}$. So for z near a, $\exists\begin{pmatrix}x\\y\end{pmatrix}$ s.t. $H\begin{pmatrix}x\\y\end{pmatrix}=\begin{pmatrix}z\\0\end{pmatrix}$... so set $g(z):=\pi_2\circ H^{-1}\begin{pmatrix}z\\0\end{pmatrix}$

* When is $DH\begin{pmatrix}a\\b\end{pmatrix}$ invertible?

$DH=\begin{pmatrix}I & 0\\ \frac{\partial f}{\partial x} & \frac{\partial f}{\partial y}\end{pmatrix}$ need $\frac{\partial f}{\partial y}$ invertible at (a,b).

* What is Dg?

$f(x,g(x))=0\implies$

$$\begin{pmatrix}\frac{\partial f}{\partial x} & \frac{\partial f}{\partial y}\end{pmatrix}\begin{pmatrix}I\\ \partial g/\partial x\end{pmatrix}=0$$

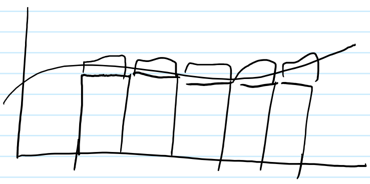
$$\text{so } \frac{\partial g}{\partial x}=-\left(\frac{\partial f}{\partial y}\right)^{-1}\frac{\partial f}{\partial x}=-\left[\frac{\partial f}{\partial y}(x,g(x))\right]^{-1}\frac{\partial f}{\partial x}(x,g(x))$$

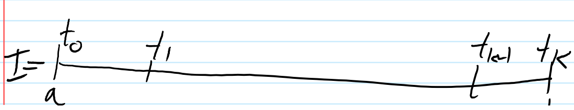
Recall overall goal: $\int_M dw=\int_{\partial M}w$

done, 11/10

$f:\mathbb{R}\to\mathbb{R}$

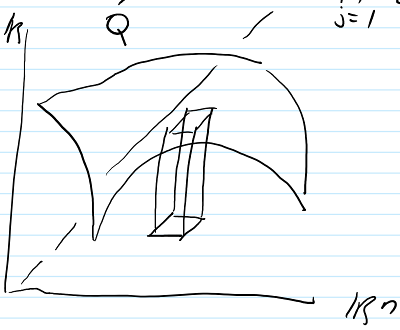
$\int_a^b f(x)\,dx=\int_{[a,b]}f$

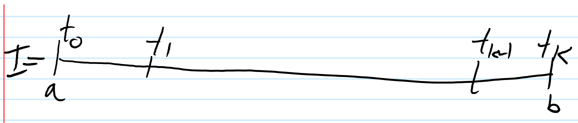




$f:\mathbb{R}^n\longrightarrow\mathbb{R}$

$\int_Q f$ $\quad Q=\prod_{j=1}^n[a_j,b_j]$





$P = (t_0, \ldots, t_k)$

or

$P = \{[t_0, t_1], \ldots [t_{k-1}, t_k]\}$

$J \in P \Leftrightarrow J$ is one of these

$\ell(J) = \ell([c,d]) = d - c$

$m_J(f) = \inf\{f(x) : x \in J\}$

$M_J(f) = \sup\{f(x) : x \in J\}$

$L(f, P) = \sum_{J \in P} m_J(f)\ell(J)$

$U(f, P) = \ldots$

$\underline{\int_{[a,b]}} f = \sup(L(f,P) : P$ a partition of $[a,b]$

$\overline{\int_{[a,b]}} \quad \ldots$

__Def__ f is integrable on $[a,b]$ if $\underline{\int_{[a,b]}} f = \overline{\int_{[a,b]}} f$ & then the common value is the "integral of f on $[a,b]$, $\int_{[a,b]} f$

$\underline{P} = (P_1 \ldots P_n)$

P_j a partition of $[a_j, b_j]$

$R = \prod [c_j, d_j] \in \underline{P}$ if $\forall j\ [c_j, d_j] \in P_j$

$V(R) = V(\prod [c_j, d_j]) = \prod_{j=1}^{n} (d_j - c_j)$

$m_R(f)$

$M_R(f)$

$L(f, \underline{P}) = \sum_{R \in \underline{P}} m_R(f) V(R)$

$\underline{\int_Q} f$