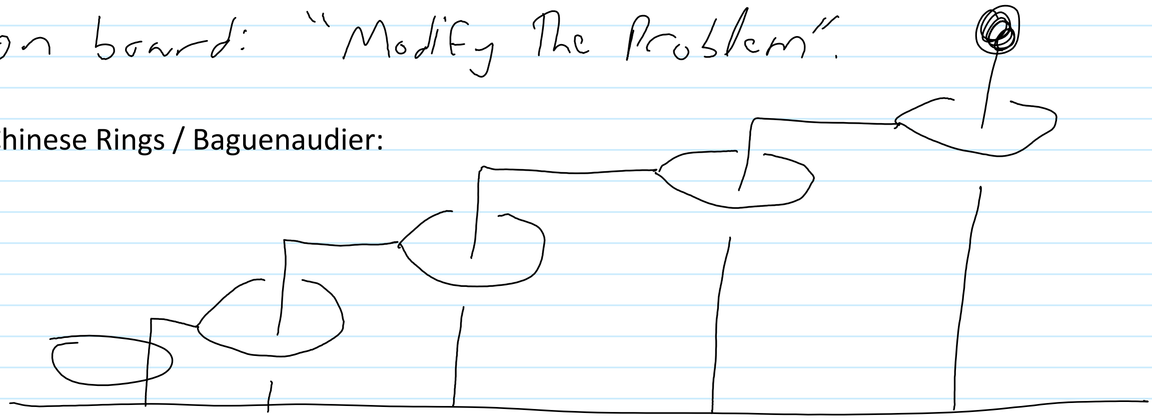
16-475 Tue Feb 2, hours 10-11: Modify the Problem / Choose an Effective Notation

February 2, 2016 10:57 AM

on board: "Modify The Problem".

Chinese Rings / Baguenaudier:



1.4.3. Prove that there do not exist positive integers x, y, z such that

$$x^2 + y^2 + z^2 = 2xyz.$$

1.4.4. Evaluate $\int_0^\infty e^{-x^2}\,dx$.

a classic.

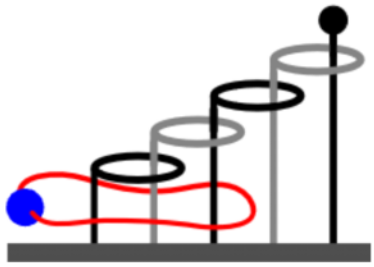
modify to $x^2+y^2+z^2 = (\text{even})\,xyz$
& consider all possible parities.

Handout for February 5, "Modify the Problem" / "Choose an Effective Notation"

Reading. Sections 1.4 and 1.5 of Larson's textbook.

Next Quiz. On Thursday February 4, mostly problems from this handout and from Larson's Sections 1.4 and 1.5.

Problem 0. Solve the "Chinese Rings", or "Baguenaudier" on the right.



Problem 1 (Larson's 1.4.4). Compute $\int_0^\infty e^{-x^2}dx$.

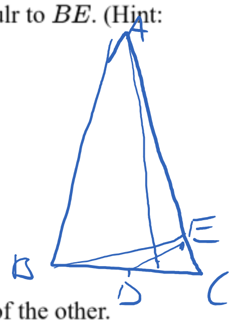
Problem 2. Compute the volume of the n-dimensional sphere $S^n := \{x \in \mathbb{R}^{n+1} \colon |x| = 1\}$ in $\mathbb{R}^{n+1}$ and the volume of the n-dimensional ball $D^n := \{x \in \mathbb{R}^n \colon |x| \leq 1\}$ in $\mathbb{R}^n$.

Problem 3 (Larson's 1.5.1). One morning it started snowing at a heavy and constant rate. A snowplow started out at 8:00AM. At 9:00AM, it had gone 2km. By 10:00AM, it had gone 3km. Assuming that the snowplow removes a constant volume of snow per hour, determine the time at which it started snowing.

Problem 4 (Larson's 1.5.2).

1. If $n \in \mathbb{N}$ and $2n+1$ is a square, show that $n+1$ is the sum of two successive squares.
2. If $n \in \mathbb{N}$ and $3n+1$ is a square, show that $n+1$ is the sum of three successive squares.

Problem 5 (Larson's 1.5.3). In a triangle ABC, $AB = AC$, D is the mid point of BC, E is the foot of the perpendicular drawn from D to AC, and F is the midpoint of DE. Prove that AF is perpendiculr to BE. (Hint: use analytic geometry and be clever about the choice of coordinate system).



Problem 6 (Larson's 1.5.4). Let $-1 < a_0 < 1$ and define recursively for $n > 0$,

$$a_n = \left(\frac{1 + a_{n-1}}{2}\right)^{1/2}.$$

What happens to $4^n(1 - a_n)$ as $n \to \infty$?

Problem 7 (Larson's 1.5.6). Guy wires are strung from the top of each of two poles to the base of the other. What is the height from the ground where the two wires cross?

Problem 8. What are your favourite "Modify the Problem" and "choose an effective notation" problems?