Tuesday Feb 10, hour 16: Exploit Symmetry

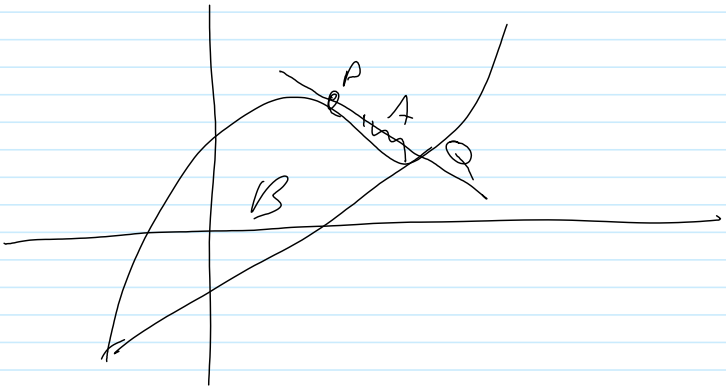
February-10-15 8:34 AM

Advance preps: Symmetry handout & ref. card. browser setup.

on board: Today's Menu: Handout P7, 5?, Explore Symmetry6, 4 also put "$50 bounty rules".

Problem 7. Prove: You cannot colour the points of the plane with just three colours, so that no two points of distance 1 will be coloured with the same colour. What if you had four colours available? Do. } on board

Problem 5 (Larson's 1.6.4). Let P be a point on the graph G of $y = f(x)$, where f is a cubic polynomial. Assume the tangent to the curve at P intersects G again at a point Q. Let A be the area bound by G and the segment PQ, and let B be the area defined in exactly the same manner, except starting with Q rather than with P. What is the relationship between A and B?



WLOG, $y = x^3 + ax$ $\quad P = (x_0, y_0)$

$Q = (-2x_0, \_\_)$ $\quad \frac{y - y_0}{x - x_0} = 3x_0^2 + a$

$A = \int_{-2x_0}^{x_0} (x^3 + ax - \dots)\,dx$ $\quad y = (3x_0^2 + a)(x - x_0) + y_0$

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In[2]:= Integrate[a t + t^3 - a x0 - x0^3 - (t - x0) (a + 3 x0^2), {t, -2 x0, x0}]

Out[2]= 27 x0^4 / 4
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Dror's Favourite "Explore Symmetry" topic: See http://drorbn.net/Treehouse:

Theorem. There are precisely 17 patterns with which to tile the plane, no more, no less. They are all made of combinations of the 10 basic features, 2, 3, 4, 6, ✌, ✋, 4̄, 6̄, M, and G, as follows:

✓	Dror's	Conway's	crystallographic	✓	Dror's	Conway's	crystallographic
	2222	2222	p2		3✌	3*3	p31m
	333	333	p3		✌✌✌	2*22	cmm
	442	442	p4		22M	22*	pmg
	632	632	p6		MM	**	pm
	~~2222~~	*2222	pmm		MG	*o	cm
	~~333~~	*333	p3m1		GG	oo	pg
	~~442~~	*442	p4m		22G	22o	pgg
	~~632~~	*632	p6m		∅	0	p1
	42	4*2	p4g				

$50 bounty for a 333!

1. "In nature" – must have been there before you came, not made for this purpose, not on web.
2. Bring a picture! You must either take it on board or be in it!

Fame also for the lesser symmetries!