Quiz 10 on March 26, 2015: “Consider Extreme Cases”. You have 25 minutes to solve two of the three problems below. Please write on both sides of the page.

Marking Comment. My decision remains to simplify the management of this course and mark the quizzes myself, though at a delay of one week, in a symbolic acknowledgement of the ongoing TA strike.

Problem 1 (Larson’s 1.11.4). Prove that the product of $n$ successive integers is always divisible by $n!$.

Problem 2 (Larson’s 1.11.2, reworded). Let $A$ be a set of $2n$ points in the plane, no three of them on the same line. Suppose that $n$ of them are coloured red and $n$ are coloured blue. Show that you can choose a pairing of the reds and the blues such the straight line segments between the pairs do not intersect.

Problem 3 (Larson’s 1.11.7). Show that there exists a rational number $c/d$, with $d < 100$, such that $\lfloor k \frac{c}{d} \rfloor = \lfloor k \frac{73}{100} \rfloor$ for $k = 1, 2, \ldots, 99$. 