15-344 Combinatorics on Sep 15 - hours 1-2: Course Introduction

Sunday, September 13, 2015 1:15 PM

on board.

Handout on side board.

MAT 344 Combinatorics

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Today's Reading: 1. Handout!

2. Textbook introductions & section 1.1.

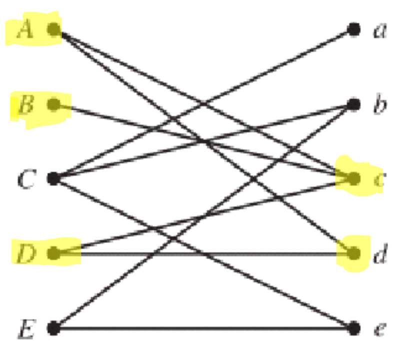
Go over Day One Handout. Highlights:

Definition A graph $G=(V,E)$ is a (usually finite) set V along with a set E of unordered pairs of distinct elements of V.

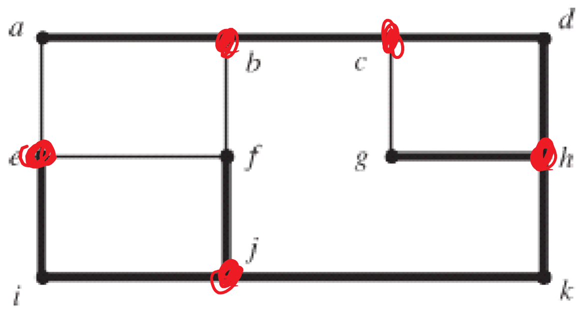
V: The "vertices" of G

E: The "edges" of G.

If $e=(ab)\in E$, we say that e is incident to a and to b.



No matching because ABD compete for cd.



Network reliability. How many edges can you remove while keeping this graph connected? How many vertices can you remove keeping the rest connected? What's the smallest connected subset of this graph?
Street surveillance. What's the smallest "edge cover" for this graph?
Scheduling. What's the smallest number of independent sets that cover this graph? What's the largest independent set in this graph?

How many edges can you remove while keeping the graph connected?

(ab), (ae) ; (bc)(jk)

---- same for vertices. } skipped

----- minimal connected subset.

Definition An "edge cover" for $G=(V,E)$ is a subset C of V such that every $e \in E$ is incident to at least one element of C.

In our graph, 14 edges, each vertex is incident to at most 3 edges $\Rightarrow |C| \geq 5$

only example in red! $[|V|=11,$ 6 trivalent, 5 bivalent$]$

Definition An "independent set of vertices" in $G=(V,E)$ is a subset $I \subset V$ s.t. $\forall a,b \in I \quad (ab) \notin E$.

How many independent sets you need to cover V?

What's the largest indep. set in V?

Theorem. "Edge covers" are complementary to "independent sets"

Theorem. "Edge covers" are complementary to "independent sets".

In other words, given $C \subset V$,
C is an edge cover iff $I = V - C$ is independent,

Proof Assume C is an edge cover. If I is not independent...

Assume I is indep. and $e = (a, b) \in E$.
Then not both $a \in I$ & $b \in I$ so at least one of a, b is in $C = V - I$.

The rest of the handout in brief...