15-344 Combinatorics on Oct 15, hour 15: More 4CT, chromatic polynomials, trees

Thursday, September 17, 2015 7:59 PM

HW4 on web.

Read Along: Sects 2.3, 2.4, 3.1.

TA Office hours: Mondays 3:30-4:30 and Tuesdays 6-7PM at 215 Huron, room 1012.

Two lies last class!

History:
First mentioned Mobius 1840.
Conjectured Guthrie 1852.
Mis-proven Kempe 1879.
Bug found Heawood 1890.
Proven Appel-Haken 1976.
Coq proof Gonthier 2005.

$P_k(G) :=$ # of k-colourings of G

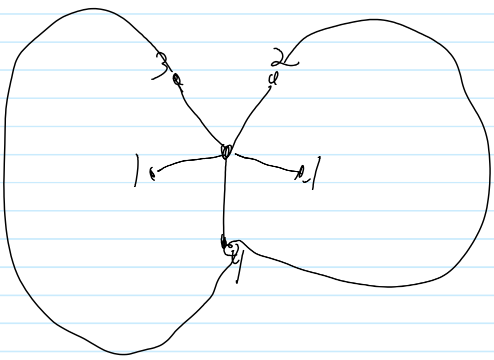
$\text{Empty}_n \longrightarrow k^n$

$L_n: \underbrace{\circ\!-\!\circ\!-\!\circ\!-\!\circ}_{n \text{ verts}} \longrightarrow k(k-1)^{n-1}$

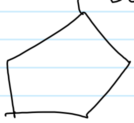
$K_n \longrightarrow k(k-1)\cdots(k-n+1)$

on board

Analyze.



3. Trees. (Connected, no circuits)

4. C_5:  } no do

Then $G_{+(x,y)}$ $G_{x=y}$ $G_{-(x,y)}$

Thm

up 1. If $(x,y) \notin E$, $P_k(G) = P_k(G_{+(x,y)}) + P_k(G_{x=y})$.

down 2. If $(x,y) \in E$, $P_k(G) = P_k(G_{-(x,y)}) - P_k(G_{x=y})$.

Do example 4.

done, though w/o proof of Thm, testing only on C_4 and only going up.

Little on trees:

<u>Thm</u> For a connected graph T, TFAE:

1. T has no circuits.
2. Let a be some vertex of T. For any other vertex x of T there is a unique path from a to x.
3. There is a unique path between any two $x \neq y \in V$
4. T is "minimally connected": it becomes disconnected upon the removal of any edge in T.

<u>Pf</u> $1 \Rightarrow 4 \Rightarrow 3 \Rightarrow 2 \Rightarrow 1$.

<u>Def</u> Such T is a tree.

<u>Thm</u> In a tree, $e = v-1$.