15-344 Combinatorics on Nov 24, hours 29-30: Generating Functions

Tuesday, November 17, 2015 9:11 AM

Evaluations responses: 2/168 https://course-evals.utoronto.ca/blue/a.aspx?l=591_1709_AAAAAAABgaM

Read Along: 6.1-6.3

Agenda. Functions $\Leftrightarrow$ sequences.

Generating functions:

seq a_k $\longrightarrow$ polynomial/power series/function:

$$f_a = \sum a_k x^k$$

Examples

1. $a_k = \binom{n}{k} \Longrightarrow f_a = (1+x)^n$
2. $a_k = 1 \Longrightarrow f_a = (1-x)^{-1}$
3. $a_k = 1$ for $0 \le k \le m$, $a_k = 0$ for $k > m$: $\frac{1-x^{m+1}}{1-x}$
4. $f_a = (1+x+x^2+x^3)^8$ $\quad a = ?_6$
5. a_k = # of way of writing k as a sum of 5 non-neg integers.

$$f_a = \left(\frac{1}{1-x}\right)^5 = \frac{1}{(1-x)^5} = \sum_{k=0}^{\infty} \binom{k+4}{4} x^k$$

in general,

$$\frac{1}{(1-x)^r} = \sum_{k=0}^{\infty} \binom{k+r-1}{r-1} x^k$$

6. If a & b are sequences, what is $f_a + f_b$?, $f_a \cdot f_b$?, What's f_a' ?, What's $x f_a'$?
7. What's a_{16} if $f_a = (x^2+x^3+x^4+\dots)^5$? skipped
8. How many ways to select 25 toys from

seven types of toys w/ 2–6 of each type.

9. Compute $\sum_{k=0}^{n} k\binom{n}{k}$.

10 Compute $\sum_{k=0}^{n} k^2\binom{n}{k}$.

Last line.

Def A "partition" of n is a way of dividing n identical objects into some number of ~~baskets~~, whose order is immaterial. p_n is the number of such partitions.

Example: $p_5 = 7$.

5
4+1
3+2
3+1+1
2+2+1
2+1+1+1
1+1+1+1+1

Q what is f_p? I.e., what is $\sum_n p_n x^n$?

Soln

$$f_p = (1+x+x^2+\dots)(1+x^2+x^4+x^6+\dots)\cdots$$

$$= \prod_{k=1}^{\infty} \frac{1}{(1-x^k)}$$