15-344 Combinatorics on Nov 19, hour 28: Combinatorial Identities, Generating Functions

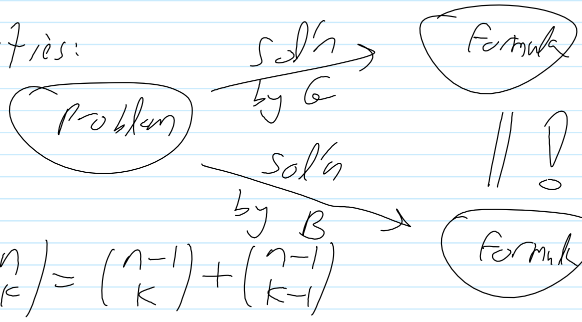
Tuesday, November 17, 2015 9:11 AM

HW8 on web by midnight!

Read Along: 5.5, 6.1.

Agenda. combinatorial identities, generating fnctns.

Comb. identities:



Examples:

$\binom{n}{k}=\binom{n}{n-k}$ $\quad$ $\binom{n}{k}=\binom{n-1}{k}+\binom{n-1}{k-1}$

Important example: $a\cdot b=b\cdot a$

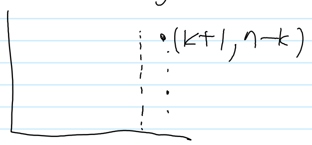
More examples:

1. $\sum\binom{n}{k}=2^n$ $\qquad$ 1. comb. 2. alg.

2. $\binom{n}{m}\binom{m}{k}=\binom{n}{k}\binom{n-k}{m-k}$ $\qquad$ 1. comb. 2. alg.

3. $\binom{k}{k}+\binom{k+1}{k}+\binom{k+2}{k}+\dots+\binom{n}{k}=\binom{n+1}{k+1}$

   1. combinatorial argument. 2. grid argument:



4. $\binom{n}{0}+\binom{n+1}{1}+\binom{n+2}{2}+\dots+\binom{n+k}{k}=\binom{n+k+1}{k}$

   (invent a story)!

5. $\sum\binom{n}{k}^2=\binom{2n}{n}$

6. $\sum_{k=0}^{r}\binom{m}{k}\binom{n}{r-k}=\binom{m+n}{r}$

The book has 2 more....

Done line

Generating functions:

seq a_k $\longrightarrow$ polynomial/power series/function:

$$f_a=\sum a_k x^k$$

Examples

1. $a_k = \binom{n}{k} \Rightarrow f_a = (1+x)^n$
2. $a_k = 1 \Rightarrow f_a = (1-x)^{-1}$
3. $f_a = (1+x+x^2+x^3)^8 \qquad a_6 = ?$
4. a_k = # of way of writing k as a sum of 5 non-neg integers.

   $f_a = \left(\frac{1}{1-x}\right)^5$