

Pensieve header: A formula for the Fibonacci numbers, plus some tidbits.

```
{λ1, λ2} = x /. Solve[x2 - x - 1 == 0]
```

```
{ $\frac{1}{2} (1 - \sqrt{5})$ ,  $\frac{1}{2} (1 + \sqrt{5})$ }
```

```
F[n_] := Expand[a λ1n + b λ2n];
```

```
Solve[F[0] == 1 ∧ F[1] == 1, {a, b}] /. Rule -> Set
```

```
{ $\{\frac{1}{10} (5 - \sqrt{5})$ ,  $\frac{1}{10} (5 + \sqrt{5})\}$ }
```

```
F[n] // Simplify
```

```
 $\frac{1}{5} 2^{-1-n} \left( - (1 - \sqrt{5})^n (-5 + \sqrt{5}) + (1 + \sqrt{5})^n (5 + \sqrt{5}) \right)$ 
```

```
F[1000]
```

```
70 330 367 711 422 815 821 835 254 877 183 549 770 181 269 836 358 732 742 604 905 087 154 537 \
118 196 933 579 742 249 494 562 611 733 487 750 449 241 765 991 088 186 363 265 450 223 647 106 \
012 053 374 121 273 867 339 111 198 139 373 125 598 767 690 091 902 245 245 323 403 501
```

```
Table[F[n], {n, 0, 10}]
```

```
{1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89}
```

```
{F[21]/F[20], GoldenRatio} // N
```

```
{1.61803, 1.61803}
```

```
n = 30; F[n]
```

```
1 346 269
```

Homework (not to be submitted). What is the story told by each of the following formulas?

```
{ $F[\frac{n}{2}]^2 + F[\frac{n}{2} - 1]^2$ ,  $\sum_{k=0}^{n/2} \text{Binomial}[n - k, k]$ ,  $\sum_{k=0}^{n-2} F[k]$ ,  $\sum_{k=0}^{n/2-1} F[n - 2k - 1]$ }
```

```
{1346269, 1346269, 1346268, 1346268}
```

```
Table[Binomial[n, k], {n, 0, 10}, {k, 0, 10}] // MatrixForm
```

```
(
1 0 0 0 0 0 0 0 0 0 0
1 1 0 0 0 0 0 0 0 0 0
1 2 1 0 0 0 0 0 0 0 0
1 3 3 1 0 0 0 0 0 0 0
1 4 6 4 1 0 0 0 0 0 0
1 5 10 10 5 1 0 0 0 0 0
1 6 15 20 15 6 1 0 0 0 0
1 7 21 35 35 21 7 1 0 0 0
1 8 28 56 70 56 28 8 1 0 0
1 9 36 84 126 126 84 36 9 1 0
1 10 45 120 210 252 210 120 45 10 1
)
```