Do not turn this page until instructed.

Math 344 Combinatorics

Term Test

University of Toronto, October 27, 2015

Solve 4 of the 5 problems on the other side of this page.
Each problem is worth 25 points.
You have an hour and fifty minutes to write this test.

Notes

- No outside material other than stationary is allowed.

- **Neatness counts! Language counts!** The *ideal* written solution to a problem looks like a page from a textbook; neat and clean and made of complete and grammatical sentences. Definitely phrases like "there exists" or "for every" cannot be skipped. Lectures are mostly made of spoken words, and so the blackboard part of proofs given during lectures often omits or shortens key phrases. The ideal written solution to a problem does not do that.
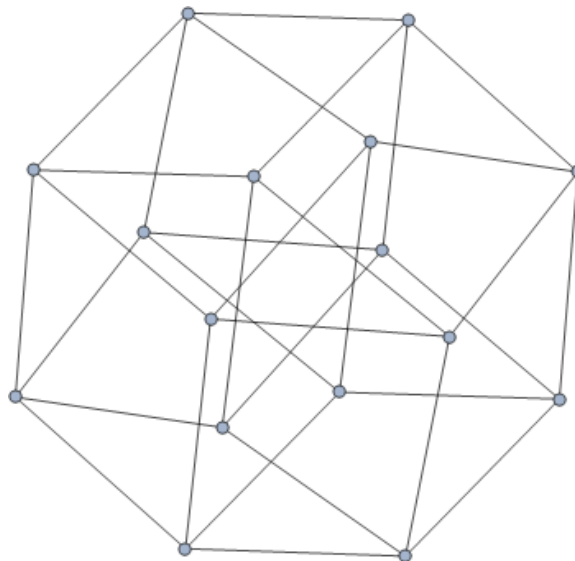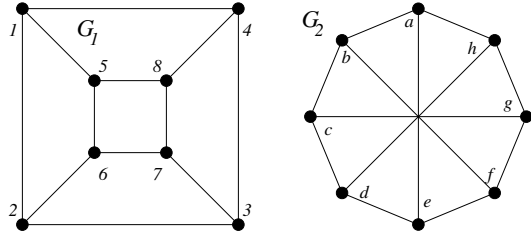

a 4D cube

Good Luck!

Solve 4 of the following 5 problems. Each problem is worth 25 points. You have an hour and fifty minutes. **Neatness counts! Language counts!**

**Problem 1.** Consider the two graphs G_1 and G_2 shown on the right.

1. For each of these graph decide whether or not it is bipartite.

2. Is G_1 isomorphic to G_2?

Tip. In math exams, yes/no answers must be accompanied with a proof.

Tip. Don't start working! Read the whole exam first. You may wish to start with the questions that are easiest for you.

Problem 2. Build the range graph for the mountain range shown on the right and use it to solve the "Mountain Climbers Puzzle" for that range.
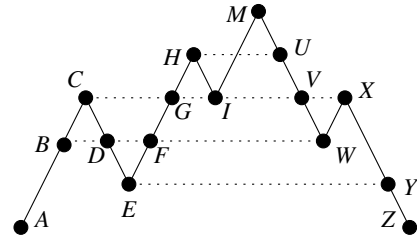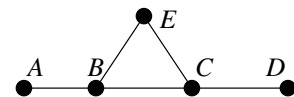
Problem 3. Consider the complete bipartite graph $K_{3,3}$.

1. Prove that it is not planar.

2. Does it have an Euler cycle or path? Justify your answer, and if you are using any theorems proven in class, state them in full.

Problem 4. Without using the four colour theorem, prove that if G is a planar graph such that every proper subgraph of G has a 4-colouring and such that G has a vertex of degree 4, then G has a 4-colouring.

Tip. X' is called "a proper subset of a set X" if $X' \subset X$ and $X' \neq X$. Likewise, G' is a proper subgraph of a graph G if $G' \subset G$ and $G' \neq G$.

Problem 5. Compute the chromatic polynomial of the "Mexican Hat" graph shown on the right.

Good Luck!