HW7 on web by midnight!

Read Along 2.1–2.3

Today: rank nullity, matrices.

(141102) Assaf's riddle: 5 ~~k~~ kids share a loot of 50 ~~n~~ in-wrapping halloween candies. The first kid proposes a way to split the loot; if it is not accepted by a strict majority (her included), she's left out and the second proposes a split, etc. How is the loot split?

Reminder. $T: V \to W$

$N(T) = \ker T = \{v : Tv = 0\}$ "null space", "kernel".

$R(T) = \operatorname{im} T = \{Tv : v \in V\}$ "range", "image"

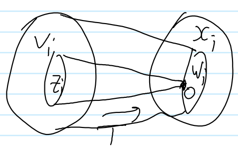
$\operatorname{nullity}(T) := \dim N(T)$ $\quad \operatorname{rank}(T) := \dim R(T)$

Thm 1 "the dimension theorem", "the rank-nullity Thm"

Given $T: V \to W$, $\underset{m}{\dim V} = \underset{r}{\operatorname{rank}(T)} + \underset{n}{\operatorname{nullity}(T)}$

PF $(z_i)_1^n$ basis of $N(T)$, extend to $(z_i) \cup (v_i)$ a basis of V,

start line



claim $w_i := T(v_i)$ are lin indep. in W PF....

claim w_i span $R(T)$ PF...

Corollary of Thm 1 If $\dim V = \dim W$, TFAE

1. T is 1-1
2. T is onto
3. $\operatorname{rank} T = \dim V$
4. T is invertible.

Thm 2 $T: V \to W$ & $T': V' \to W'$ are "isomorphic" iff $(\dim V, \dim W, \operatorname{rank} T) = (\dim V', \dim W', \operatorname{rank} T')$

i.e., $\exists$ a "commutative square of isomorphisms":

$$\begin{tikzcd} V \arrow[r, "T"] \arrow[d, "\phi"'] & W \arrow[d, "\psi"] \\ V' \arrow[r, "T'"] & W' \end{tikzcd}$$

skipable.

Reminder: Choosing a basis, V is isomorphic to F^n.

Goal: Choosing bases, $\mathcal{L}(V, W)$ is isomorphic to $M_{m \times n}$ $(m = \dim W,\ n = \dim V)$

Thm. Given V w/ basis $\beta = (v_1 \dots v_n)$
and W w/ basis $\gamma = (w_1 \dots w_m)$
We have an isomorphism

Abstract, general, coord-free — merely numbers, choice-dependent, easy to work with

$$\mathcal{L}(V, W) \longrightarrow M_{m\times n}(F)$$

$$T \longrightarrow [T]_\beta^\gamma = A$$

$$A = \begin{pmatrix} [Tv_1]_\gamma & [Tv_2]_\gamma & \cdots & [Tv_n]_\gamma \end{pmatrix} = \begin{pmatrix} a_{11} & & & a_{1n} \\ \vdots & & & \vdots \\ a_{m1} & & & a_{mn} \end{pmatrix} \Longleftrightarrow Tv_j = \sum_{k=1}^m a_{kj} w_k$$

Examples 0. 0 1. 1

done line

2. $D: P_3(\mathbb{R}) \longrightarrow P_2(\mathbb{R})$ differentiation
3. $T_\alpha : \mathbb{R}^2 \longrightarrow \mathbb{R}^2$
4. $A = \begin{pmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & & \\ a_{m1} & \cdots & a_{mn} \end{pmatrix} \rightsquigarrow L_A : F^n \to F^m$
by $L_A \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix} = \begin{pmatrix} a_{11}x_1 + \dots a_{1n}x_n \\ \vdots \\ a_{m1}x_1 + \dots a_{mn}x_n \end{pmatrix}$

$$(L_A)_{(e_i)_{i=1}^n}^{(e_j)_{j=1}^m} = A$$