Two players alternate drawing cards from the above deck. The first player to have 3 cards that add up to 15, wins. Would you like to be the first to move or the second? (More on today's web, including a video link).

https://media.library.utoronto.ca/play.php?DJ6CPFxByy2J&id=8503&access=public

HW on web.
Term test discussion & return @ 9:50.

Today: Linear transformations abstractly.

Remind: \( V, W / F \) \( L : V \to W \) "linear transformation"
if it preserves structure:

\[
L(\sum_i u_i) = \sum_i L(u_i)
\]

\( L(u, v) \) said is a vector space.
* The composition of l.t. is a l.t.
* Not commutative!
* A l.t. is determined by its values on a basis, and these values are arbitrary.

Def: \( V \) & \( W \) are isomorphic if

\( \exists \) l.t. \( R : V \to W \) and \( L : W \to V \)

\( \text{st. } L \circ R = \text{Id}_W \) & \( R \circ L = \text{Id}_V \)

Thm: If \( V, W \) are f.d. over \( F \),

then \( \dim V = \dim W \) iff \( V \) is isomorphic to \( W \).

Corollary: If \( \dim V = n \) over \( F \),

\( V \) is isomorphic to \( F^n \).

Two "mathematical structures" are "isomorphic" if there's a bijection (1-1 & onto cares) between their elements which preserves all relevant relations.

Example: Plastic chess is iso. to ivory chess, but not to checkers.

Example: The game of 15.
Def: \( N(T) = \ker T = \{ v :Tv = 0 \} \) \( \text{"null space", \"kernel\"} \)
\( R(T) = \text{im } T = \{ Tv : v \in V \} \) \( \text{"range", \"image\"} \)

Prop/Def: \( N(T) \subset V \) is a subspace; \( \text{nullity}(T) = \dim N(T) \)
\( R(T) \subset W \) is a subspace; \( \text{rank}(T) = \dim R(T) \)

Examples: \( O, I, D: P_n(\mathbb{R}) \rightarrow P_n(\mathbb{R}) \)

Thm/"the dimension theorem","the rank-nullity Thm"

Given \( T: V \rightarrow W \), \( \dim V = \text{rank}(T) + \text{nullity}(T) \)

Let \( \{ z_i \} \), basis of \( N(T) \), extend to \( \{ z_i \} \cup \{ v_i \} \) a basis of \( V \),
\( \text{claim } w_i := T(v_i) \text{ are \( \text{indep. \ in } W \), } \text{ref} \ldots \)
\( \text{claim } w_i \text{ span } R(T) \) \( \text{ref} \ldots \)