Vitamins/HV3 handout.

HW2: by Friday 5 PM @ labeled mailboxes, SS 1071, Math Aid Center.
Office Hours. Today 3-4.
Read Along 1.3–1.6
Riddle Along

Today: Subspaces, linear combinations, lin. independence.

Reminder Def: WC V is a "subspace" if it is a vector space with the operations it inherits from V.
This WC V is a subspace iff it is non-empty, closed under addition and under multiplication by a scalar.

Examples. 1. If \( A \in M_{n \times n}(F) \): \( A^t = A \)  
2. If \( A \in M_{n \times n}(F) \): \( \text{tr} A = 0 \)
3. If \( W_1 \) & \( W_2 \) are subspaces of \( V \), then so is \( W_1 \cap W_2 \) (What about \( W_1 \cup W_2 \)?)

Goal: Every \( V \) is has a "basis", so while we don't have to use coordinates, we can.

Def: \( U \) is a L.C. of \( u_1, \ldots, u_n \) if \( \text{span} u \text{ is } \text{if } u_1, u_2, \ldots, u_n \).

Examples 1. Vitamins a.s. in the handout 2. In \( P_3(\mathbb{R}) \), \( 2x^3 - 2x^2 + 12x - 6 \) is a L.C. of \( x^3 - 2x^2 - 5x - 3 \) and \( 3x^3 - 5x^2 - 4x - 9 \) but \( 3x^3 - 2x^2 + 7x + 8 \) isn't.

Then if \( \text{Span} V \text{ then } W = \text{Span}(u) = \{x \in V | x \text{ is a L.C. of } u \} \).
is a subset of \( V \).

**Def.** \( S \subseteq V \) "generates" or "spans" \( V \). (first requirement)

**Examples.** In \( V = \mathbb{R}^2 \), \( M_1 = (1, 0) \), \( M_2 = (0, 1) \), \( M_3 = (0, 0) \).

\( M_1 = (1, 0) \) \( N_1 = (0, 1) \) \( N_2 = (1, 0) \) \( N_3 = (1, 1) \). Then \( M_1 \ldots M_4 \) \& \( N_1 \ldots N_3 \) generate \( V \); but \( M_1 \ldots M_3 \) \& \( N_1 \ldots N_3 \) do not.

\( (N_1, N_2, N_3 \) are in \( \{(a, b, c) : a + b + c = 2d \} \)

\((1, 1) (1, 0) (0, 1)\)

**Def.** A subset \( S \subseteq V \) is "lin. dep." if it is "wasteful."

I.e., if \( \exists \ a _ i \) \& \( F \) not all 0 \& \( \exists \ a _ i \) s.t. \( \sum \_ a _ i w = 0 \).

Otherwise, it is "lin. indp."

**Examples.** \( \{ (0, 0) \} \quad \{ (\frac{1}{2}) , (\frac{1}{2}) , (\frac{1}{2}) \} \)

**Comm.** 1. \( \emptyset \) is lin. indp.

2. \( \{ u \} \) is lin. indp. if \( u \neq 0 \).

3. Suppose \( S_1 \subseteq S_2 \subseteq V \). Then

   a. If \( S_1 \) is dep., so is \( S_2 \).
   b. If \( S_2 \) is indp., so is \( S_1 \).

4. If \( S \) is lin. indp. in \( V \) and \( U \subseteq V \), then \( S \cup U \) is lin. dep. iff \( U \subseteq \text{Span}(S) \).