

240Algebra-141001 Hours 11-12: Subspaces, linear combination

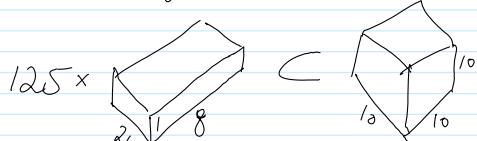
September-18-14 6:28 PM

Vitamins / HW 3 handout.

HW 2: by Friday 5PM @ labeled mailboxes, SS 1071 "Math Aid Centre".
Office Hours. Today 3-4.

Read Along 1.3 - 1.6

Riddle Along



Today: Subspaces, linear combinations, lin. independence.

Reminder Def $W \subset V$ is a "subspace" if it is a vector space

with the operations it inherits from V .

Thm $W \subset V$ is a subspace iff it is non-empty & "closed under addition and under multiplication by a scalar"

Examples 1. $\{A \in M_{n \times n}(\mathbb{F}) : A^T = A\}$

2. $\{A \in M_{n \times n}(\mathbb{F}) : \text{tr } A = 0\}$

3. If W_1 & W_2 are subspaces of V ,

then so is $W_1 \cap W_2$ (what about unions?)

Goal: Every V.s. has a "basis". So while we don't have to use coordinates, we can.

Def U is a l.c. of u_1, \dots, u_n

if $\exists \alpha_i \in \mathbb{F}$ st. $u = \sum \alpha_i u_i$

Examples 1. Vitamins as in the handout

2. In $P_3(\mathbb{R})$, $2x^3 - 2x^2 + 12x - 6$ is

a l.c. of $x^3 - 2x^2 - 5x - 3$

and $3x^3 - 5x^2 - 4x - 9$

but $3x^3 - 2x^2 + 7x + 8$ isn't.

Thm If $\{u_i\} \subset V$ then $W = \text{span}(u_i) := \{ \text{all l.c. of the } u_i \}$

One-on-One Classes: 2014-15 Math 240 Algebra I Chapter 1

Example 1 TABLE 1.1 Vitamin Content of 100 Grams of Various Foods

	Food	vitamin A (mg)	vitamin B (mg)	vitamin C (mg)	vitamin D (mg)
Sampled		0.00	0.00	0.00	0.00
Sampled (doubly homogenized)		0.00	0.00	0.00	0.00
Sampled (homogenized)		0.00	0.00	0.00	0.00
Chlorophyll (doubly homogenized)		1.00	0.00	0.10	0.10
Chlorophyll (homogenized)		0.00	0.00	0.10	0.10
Crab meat (homogenized)		0.00	0.00	0.10	0.10
Dried apricots		10.00	0.00	0.00	0.00
Orange juice (doubly homogenized)		10.00	0.00	0.00	0.00
Orange juice (homogenized)		10.00	0.00	0.00	0.00
Raw carrots, raw		0.00	0.00	0.10	0.00
Raw carrots, cooked		0.00	0.00	0.10	0.00
Salad greens, raw		0.00	0.00	0.00	0.00
Salad greens, cooked		0.00	0.00	0.00	0.00
Skinned mussels (homogenized)		0.00	0.00	0.00	0.00
Skinless chicken breast, raw		0.00	0.00	0.00	0.00
Skinless chicken breast, cooked		0.00	0.00	0.00	0.00
Spinach, raw		0.00	0.00	0.00	0.00
Spinach, cooked		0.00	0.00	0.00	0.00
Strawberries, raw		0.00	0.00	0.00	0.00
Strawberries, cooked		0.00	0.00	0.00	0.00
Tomato juice (homogenized)		0.00	0.00	0.00	0.00
Tomato juice (doubly homogenized)		0.00	0.00	0.00	0.00
Turnip greens, raw		0.00	0.00	0.00	0.00
Turnip greens, cooked		0.00	0.00	0.00	0.00
Whole milk (homogenized)		0.00	0.00	0.00	0.00
Whole milk (doubly homogenized)		0.00	0.00	0.00	0.00
Yogurt, plain		0.00	0.00	0.00	0.00

Table 1.1 shows the vitamin content of 100 grams of 12 foods with respect to vitamins A, B₁, B₂ (thiamine), B₆ (riboflavin), choline, and C (ascorbic acid).

The vitamin content of 100 grams of each food can be recorded as a column vector in \mathbb{R}^6 – for example, the column for apple juice is

$$\begin{pmatrix} 0.00 \\ 0.00 \\ 0.00 \\ 0.00 \\ 0.00 \\ 0.10 \end{pmatrix}$$

From: R. H. Martin, R. West and B. B. Woodward, *Principles of Food Hygiene and Nutrition*, 2nd edn., Chapman and Hall, London, 1972, p. 482.

*Note to nutritionists: note that the amount of a vitamin present in either one of the foods may not be the same, and will vary, as we see that

$$\begin{pmatrix} 0.00 \\ 0.00 \\ 0.00 \\ 0.00 \\ 0.00 \\ 0.00 \end{pmatrix} = \begin{pmatrix} 0.00 \\ 0.00 \\ 0.00 \\ 0.00 \\ 0.00 \\ 0.00 \end{pmatrix} + \begin{pmatrix} 0.00 \\ 0.00 \\ 0.00 \\ 0.00 \\ 0.00 \\ 0.10 \end{pmatrix} = \begin{pmatrix} 0.00 \\ 0.00 \\ 0.00 \\ 0.00 \\ 0.00 \\ 0.10 \end{pmatrix}$$

These two vectors represent the same food, but different ways of preparing it.

Thus, the columns of Table 1.1 are not unique representations of the same food.

For more details, see page 28-29.



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Table 1.2 shows the vitamin content of 100 grams of 12 foods with respect to vitamins A, B₁, B₂ (thiamine), B₆ (riboflavin), choline, and C (ascorbic acid).

The vitamin content of 100 grams of each food can be recorded as a column vector in \mathbb{R}^6 – for example, the column for apple juice is

$$\begin{pmatrix} 0.00 \\ 0.00 \\ 0.00 \\ 0.00 \\ 0.00 \\ 0.10 \end{pmatrix}$$

From: R. H. Martin, R. West and B. B. Woodward, *Principles of Food Hygiene and Nutrition*, 2nd edn., Chapman and Hall, London, 1972, p. 482.

*Note to nutritionists: note that the amount of a vitamin present in either one of the foods may not be the same, and will vary, as we see that

$$\begin{pmatrix} 0.00 \\ 0.00 \\ 0.00 \\ 0.00 \\ 0.00 \\ 0.00 \end{pmatrix} = \begin{pmatrix} 0.00 \\ 0.00 \\ 0.00 \\ 0.00 \\ 0.00 \\ 0.00 \end{pmatrix} + \begin{pmatrix} 0.00 \\ 0.00 \\ 0.00 \\ 0.00 \\ 0.00 \\ 0.10 \end{pmatrix} = \begin{pmatrix} 0.00 \\ 0.00 \\ 0.00 \\ 0.00 \\ 0.00 \\ 0.10 \end{pmatrix}$$

These two vectors represent the same food, but different ways of preparing it.

Thus, the columns of Table 1.2 are not unique representations of the same food.

For more details, see page 28-29.

is a subspace of V .

Def $S \subset V$ "generates" or "spans" V . (First requirement from "a basis")

Examples In $V = M_{2 \times 2}(\mathbb{R})$ $M_1 = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$ $M_2 = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$ $M_3 = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$

$$M_4 = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \quad N_1 = \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix} \dots \quad N_4 = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}, \text{ Then}$$

$M_1 \dots M_4$ & $N_1 \dots N_4$ generate V , but
 $M_1 \dots M_3$ & $N_1 \dots N_3$ do not.

done

Aside: If
 $S_1 \subset \text{Span}(S_2)$
then
 $\text{Span}(S_1) \subset \text{Span}(S_2)$

(N_1, N_2, N_3 are in $\left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} : a+b+c=2d \right\}$)
 $\begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}$)

Def A subset $S \subset V$ is "lin. dep" if it is "wasteful".

I.e., If $\exists a_i \in \mathbb{F}$ not all 0 $\nabla u_i \in S$ s.t. $\sum a_i u_i = 0$.

Otherwise, it is "lin. indep."

Examples $\left\{ \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 2 \\ 3 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \right\}$

Comments 1. \emptyset is lin. indep.

2. $\{u\}$ is lin. indep iff $u \neq 0$.

3. Suppose $S_1 \subset S_2 \subset V$. Then

a. IF S_1 is dep, so is S_2

b. IF S_2 is indep, so is S_1

4. If S' is lin. indep in V and $V \subset V'$, then

$S' \cup \{v\}$ is lin. dep. iff $v \in \text{Span}(S')$.