Class photo on web! HW3 due by midnight!
Office Hours: Wed 3-4 this week & next. HW2 due date?
Read Along: Sections 1.1-1.4 of textbook.
Riddle Along: \( V = \mathbb{R}^n \)

Today: Vector spaces, subspaces

Reminder: A vs. over a field \( F \) is a set \( V \),
with a special element \( 0 \), a binary +: \( V \times V \to V \)
and a binary \( \cdot \): \( F \times V \to V \), s.t.

VS1. \( x + y = y + x \) VS2. Assoc.
VS3. 0 VS4. -
VS5. 1 \( x = x \) VS6. \( a(bx) = (ab)x \)
VS7. \( a(x+y) = a(x) + a(y) \) VS8. \( (a+b)x = ax + bx \)

Examples: 1. \( F^n \)
2. \( M_{mn}(F) \)
3. \( F \setminus \{ 0, F \} \) is a set \( \); bytes / bits
4. Polynomials
5. \( \mathbb{C}/\mathbb{R} \) \( \mathbb{R}/\mathbb{Q} \) "Galois theory"

Thm 1. Cancellation law: additive, 2x multiplicative.

2. 0v is unique
3. negatives are unique.
5. \( 0 \cdot x = 0 \)
7. \( -a)x = -(a \cdot x) = a(-x) \)
8. \( CV = 0 \iff C = 0 \) v \( = 0 \)

Def: \( W \subseteq V \) is a "subspace" if it is a vector space
with the operations it inherits from \( V \) done line
They \( W \subset V \) is a subspace iff it is non-empty and closed under addition and under multiplication by a scalar.

**Examples:**
1. \( \forall A \in M_{n \times n}(\mathbb{F}) \): \( A^T = A \)
2. \( \forall A \in M_{n \times n}(\mathbb{F}) \): \( \text{tr} \ A = 0 \)
3. If \( W_1 \) \& \( W_2 \) are subspaces of \( V \), then \( W_1 \cap W_2 \) (What about unions?)

**Goal:** Every \( V \)-s. has a "basis". So while we don't have to use coordinates, we can.

Def: \( U \) is a l.c. of \( u_1, \ldots, u_n \) if \( \exists \) \( \left\{ a_i \right\} \) st. \( \sum a_i u_i = u \)

**Examples:**
1. Vitamins as in the handout.
2. In \( \mathbb{R}_3 \) (IR), \( 2x^3 - 2x^2 + 2x - 6 \) is a l.c. of \( x^3 - 2x^2 - 5x - 3 \) and \( 3x^3 - 5x^2 - 4x - 9 \)
   but \( 3x^3 - 2x^2 + 7x + 8 \) isn't.

**Thm:** If \( \{ u \} \subset V \) then \( W = \text{span}(u) = \{ \text{all l.c. of the } u \} \) is a subspace of \( V \).