Class photo today?

Office Hours: Wed 3-4 this week & next.

Read Along: Sections 1.1-1.4 of textbook.

Riddle Along: \[ V_L = 4V_S \]

Today: Vector spaces, subspaces

Reminder: A V.S. over a field \( F \) is a set \( V \), with a special element over \( V \), a binary \( +: V \times V \rightarrow V \) and a binary \( \cdot: F \times V \rightarrow V \), s.t.:

VSI: \( x + y = y + x \)

VS2: Associative

VS3: \( 0 \)

VS4: \(-\)

VS5: \( 1 \cdot x = x \)

VS6: \( a(bx) = (ab)x \)

VS7: \( a(x + y) \)

VS8: \((a + b)x \)

Examples:

1. \( F^n \)

2. \( \text{Matrix}(F) \)

3. \( F \{S, F) \) s a set j bytes/bits

4. Polynomials

5. \( \mathbb{C}/\mathbb{R} \)

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Theorem:

1. Cancellation law: additive, 2x multiplicative.

2. \( 0 \) is unique

3. Negatives are unique.

4. \( 0 \cdot x = 0 \)

5. \( a \cdot 0 = 0 \)

6. \( (a x) = a(\cdot x) = a(-x) \)
b. \( CV = 0 \iff C = 0 \lor V = 0 \)

Def: \( W \subseteq V \) is a "subspace" if it is a vector space with the operations it inherits from \( V \).

Thus \( W \subseteq V \) is a subspace iff it is non-empty and "closed under addition and under multiplication by a scalar."

Examples:
1. \( A \in \text{Mat}_{n \times n}(F): A^t = A \)
2. \( A \in \text{Mat}_{n \times n}(F): \text{tr} A = 0 \)
3. If \( W_1 \) \& \( W_2 \) are subspaces of \( V \), the so is \( W_1 \cap W_2 \) (What about unions?)

Goal: Every \( V \subseteq F \) has a "basis". So while we don't have to use coordinates, we can.

Def: \( \{ u_1, ..., u_n \} \) is a l.c. of \( V \) if \( \exists \alpha_i \in F \) s.t. \( u = \sum\alpha_i u_i \)

Examples:
1. Vitamins as in the handout
2. In \( \mathbb{P}_3(K) \), \( 2x^3 - 2x^2 + 12x - 6 \) is a l.c. of \( x^3 - 2x^2 - 5x - 3 \) and \( 3x^3 - 5x^2 - 4x - 9 \)
   but not \( 3x^3 - 2x^2 + 7x + 8 \) is.

Then if \( \{ w_1, ..., w_n \} \) \( \subseteq V \) then \( W = \text{span}(w_i) := \{ \alpha_i w_i \mid \alpha_i \in F \} \) is a subspace of \( V \).