

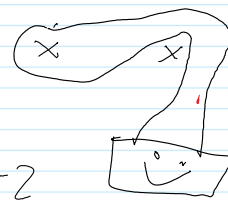
Pre-class: Fix up

[http://drorbn.net/index.php?title=14-240/Classnotes\\_for\\_Monday\\_September\\_15](http://drorbn.net/index.php?title=14-240/Classnotes_for_Monday_September_15)

Lost a pencil case a- MP202? Ask Dror.

Read Along. Appendices A-D, Sections 1.1-1.2.

Riddle Along. Can you hang a picture on two nails, so that if you remove any one of them, the picture falls? The same with 3?



Today's menu. The complex numbers, vector spaces.

- Reminder:  $\exists!$   $\tau: \mathbb{Z} \rightarrow F$  s.t.
1.  $\tau(0) = 0$   $\tau(1) = 1$
  2.  $\tau(m+n) = \tau(m) + \tau(n)$
  3.  $\tau(mn) = \tau(m)\tau(n)$

Def characteristic.

previous theme: "abstraction, generalization, definitions, examples, properties"  
next: "dream, implications, formalization & proof"

Dream Add to  $\mathbb{R}$  some number  $i$  so that  $i^2 = -1$

Implications must add  $7i, 3-7i, (2+3i)(3-7i), (3-7i)^{-1}, \dots$

Formally define  $\mathbb{C}$  and verify fieldness.

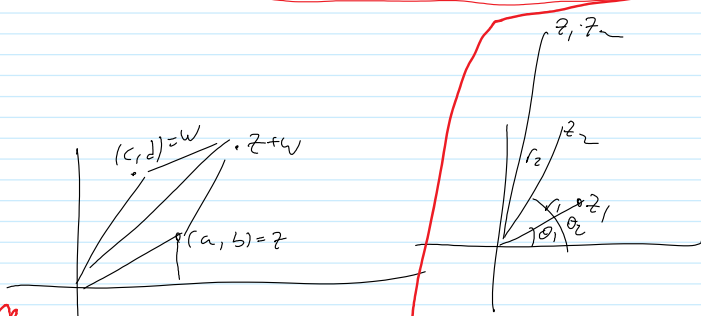
Thm Our definitions indeed make a field!

More fully: There exists a field  $\mathbb{C}$  that contains the real numbers and also contains an element  $i$  s.t.  $i^2 = -1$ .

show "The Complex Numbers by Computer"

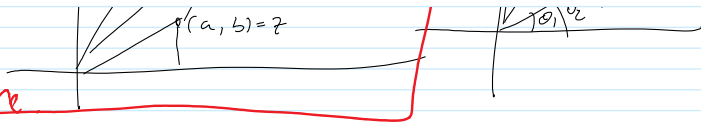
2012  
line (how 5)

Interpretation



2-14 hour 6 line

2014 hour 6 live



Waves, AC, RLC

$$V = RI$$

$$V = I/C$$

Why aren't we also adding  $\sqrt{i}$ ?

vector spaces.

Motivation: Forces can be added and multiplied by scalars

Def Let  $F$  be a Field. A v.s. over  $F$  is a set  $V$ , with a special element  $0 \in V$ , a binary  $+$ :  $V \times V \rightarrow V$  and a binary  $\cdot$ :  $F \times V \rightarrow V$ , s.t.

2012 hour 6  
live

- VS1.  $x+y = y+x$       VS2: Assoc.
- VS3.  $0$                       VS4:  $-$
- VS5:  $1 \cdot x = x$               VS6  $a(bx) = (ab)x$
- VS7  $a(x+y)$               VS8  $(a+b)x$

Examples: 1.  $F^n$

- 2.  $M_{m \times n}(F)$
- 3.  $\mathcal{F}(S, F)$        $S$  a set
- 4. Polynomials
- 5.  $\mathbb{C}/\mathbb{R}$        $\mathbb{R}/\mathbb{Q}$       "Galois theory"

Thm 1. Cancellation law: additive,  $2 \times$  multiplicative.

- 2.  $0_V$  is unique
- 3. negatives are unique.
- 5.  $0 \cdot x = 0$       6.  $a \cdot 0 = 0$
- 7.  $(-a)x = -(ax) = a(-x)$
- 8.  $cV = 0 \Rightarrow c = 0 \vee V = 0$