Understanding

P onto, Q 1–1 $\implies \operatorname{rank}(T) = \operatorname{rank}(PTQ)$:   With $A = [T]_\beta^\gamma$, $\operatorname{rank}(A) = \operatorname{rank}(T)$:

$$
\begin{array}{ccc}
V & \xrightarrow{\;T\;} & W \\
Q \uparrow & & \downarrow P \\
V' & \xrightarrow[PTQ]{\;T'\;} & W'
\end{array}
\qquad\qquad
\begin{array}{ccc}
V & \xrightarrow{\;T\;} & W \\
[\ ]_\beta \downarrow & & \downarrow [\ ]_\gamma \\
\mathbb{F}^n & \xrightarrow[T_A]{\;A\;} & \mathbb{F}^m
\end{array}
$$

High school. "Understanding is a painful process. You only understand by doing many, many routine exercises."

Undergrad. "Understanding is a slow process. You only really understand a class once you take its continuation class."

In both cases understanding is a passive thing. You do your thing and it comes when it wants.

What I learned only at the very end of my undergraduate degree is to turn "understanding" into a deliberate, active process. I still don't know how to explain it to others, yet if you think hard enough about a subject and truly try to decipher it, eventually you can - here and now, and not next year.

In fact, that process of "understanding" is not very different, at least in my mind, from the process called "research".

(not distributed)