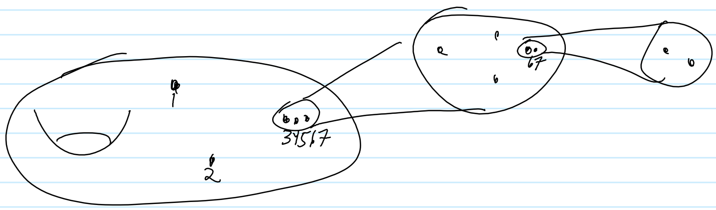
Wednesday-6 AKT on 140212: The Fulton-MacPherson compactification, 2

February-05-14 8:34 AM

Happy birthday Iva!



$$C_A(M) := \coprod_{\{A_1,\dots,A_k\},\ A=\cup A_\alpha} \left\{ \left(p_\alpha \in M, c_\alpha \in \tilde{C}_{A_\alpha}(T_{p_\alpha}M)\right)_{\alpha=1}^k : p_\alpha \neq p_\beta \text{ for } \alpha \neq \beta \right\}$$

where if V is a vector space and A is a singleton, $\tilde{C}_A(V) := \{\text{a point}\}$ and if $|A| \geq 2$,

$$\tilde{C}_A(V) := \coprod_{\substack{\{A_1,\dots,A_k\} \\ A=\cup A_\alpha;\ k\geq 2}} \left\{ \left(v_\alpha \in V, c_\alpha \in \tilde{C}_{A_\alpha}(T_{v_\alpha}V)\right)_{\alpha=1}^k : v_\alpha \neq v_\beta \text{ for } \alpha \neq \beta \right\} \Big/ \text{translations and dilations.}$$

acting on the v's.

Every "grouping" loses one dimension.

"A manifold w/ corners"

start line

Thm 1. M compact $\Rightarrow C_A(M)$ compact.

2. Singletons & doubletons.

3. $B \subset A \Rightarrow \exists\, P_B : C_A(M) \to C_B(M)$. In particular,

$\exists\, \phi_{ij} : C_A(\mathbb{R}^n) \longrightarrow C_{\{i,j\}}(\mathbb{R}^n) \sim S^{n-1}$

4 If $f: M \to N$ is a smooth embedding,

$\exists\, f_* : C_A(M) \longrightarrow C_A(N)$

Skip section in handout about $C_D(M)$; just write $C^o_D(M) := \{p\colon A \to M\colon p(a_0) \neq p(a_1) \text{ whenever } a_0 \overset{D}{\text{——}} a_1\}$

Definition 9. Write $S^n = \mathbb{R}^n \cup \{\infty\}$ and set $\bar{C}_A(\mathbb{R}^n) := \left\{c \in \bar{C}_{A\cup\{\infty\}}(S^n)\colon p_\infty(c) = \infty\right\}$.

Theorem 10. $\bar{C}_A(\mathbb{R}^n)$ is a compact manifold with corners and the direction maps $\phi_{ij}\colon \bar{C}_A(\mathbb{R}^n) \to S^{n-1}$ remain well-defined.

Finally, given $\gamma\colon S^1 \to \mathbb{R}^3$ and disjoint finite sets A and B, we set

$$C^\gamma_{A,B} := \left\{(c', c)\colon c' \in C_A(S^1),\ c \in \bar{C}_{A\cup B}(\mathbb{R}^3),\ \gamma_*(c') = p_A(c)\right\}$$

(and similarly C^γ_D for appropriate graphs D). The obvious variants of the theorems remain valid.

done line.

A word about signs.

$\mathcal{D}^{-1} = \left\langle \begin{array}{l}\text{v.s. spanned by connected} \\ \text{trivalent } D\text{'s with skeleton } S^1, \\ \text{oriented edges \& ordered } V_i \text{ \& } V_s\end{array} \right\rangle \Big/$

For internal edges: $\longrightarrow + \longleftarrow = 0$

re-ordering V_i / V_s acts by the sign of the permutation

$\mathcal{D}^0$

\ oriented edges & ordered V_i & V_s / acts by the sign of the permutation

$$\text{Lemma}\quad \mathcal{D}^{-1} \simeq \left\langle \bigcirc \right\rangle \Big/ \ldots + \ldots = 0$$

$\mathcal{D}^0$

Trivalent connected with skeleton S^1, unoriented internal edges, unordered V_i, V_s, but "oriented internal verts?"

$$\partial C_D^\gamma = \underbrace{\bigcup_{\text{edges of } D} (S^2 \text{ or } \ldots) \times C^\gamma_{D/e}}_{\text{principal faces}} \cup \underbrace{\bigcup_{\text{bigger subdiagrams}} \text{mess}}_{\text{hidden faces (wishful thinking)}}$$

3. Our diagrams: $\mathcal{D}_n^m = \left\{ \pm \bigotimes : \begin{array}{l} m = \sum m_i - 3 = 2|E| - 3|V| \\ n = -\chi = |E| - |V| \end{array} \right\} \Big/ \text{signs}$

directed internal edges, ordered -"- vertices

Our space: $\Gamma : \{\text{all embeddings } \gamma : S^1 \to \mathbb{R}^3\}$

our map: $I : \mathcal{D}_n^m \to \Omega^m(\Gamma)$ by $I(D) = \pi_* \Phi_D^* W^{|E|}$

wishful thinking: $(\partial\pi)_* \Phi_D^* W^{|E|}$ vanishes on hidden faces

Conclusion:

$$\begin{array}{ccccccc} \cdots \longrightarrow & \mathcal{D}_n^m & \xrightarrow{d} & \mathcal{D}_n^{m+1} & \longrightarrow \cdots \\ & \downarrow I & & \downarrow I & \\ \cdots \longrightarrow & \Omega^m(\Gamma) & \xrightarrow{d} & \Omega^{m+1}(\Gamma) & \longrightarrow \cdots \end{array}$$

Post Morton (March 11, 2014):

I should have accumulated this and the following few Wednesday into a single handout, "Configuration Space Integral for Knots".