Bhattacharyya theorem.

\[ Z_M(Y) = \sum_{D \in \text{Aut}(D)} \int \Phi_{\mathbb{D}} W \in \mathcal{D} \cdot \frac{1}{|\text{Aut}(D)|} \cdot C_{\mathbb{D}(1R^2, 1)}(s_1)(1R^2, 1) \cdot \text{board} \]

Let \( M \) be a \( d \)-manifold and \( A \) a point set.

\[ C^0_A(M) = \left\{ \text{injections } p: A \rightarrow M \right\}, \quad \dim C^0_A(M) = |A| \cdot d \]

\[ C_A(M) := \prod_{\{A_1, ..., A_k\}} \left\{ (p_\alpha \in M, c_\alpha \in C_A(T_{p_\alpha}M))_{\alpha=1}^{|A|} : p_\alpha \neq p_\beta \text{ for } \alpha \neq \beta \right\} \]

where if \( V \) is a vector space and \( A \) is a singleton, \( C_A(V) := \{ \text{a point} \} \) and if \( |A| \geq 2 \),

\[ C_A(V) = \prod_{\{A_1, ..., A_k\}} \left\{ (v_\alpha \in V, c_\alpha \in C_A(T_{p_\alpha}V))_{\alpha=1}^{|A|} : v_\alpha \neq v_\beta \text{ for } \alpha \neq \beta / \text{translations and dilations}, \text{ acting on the V.S.} \right\} \]

"big cell" is at \( \dim = d \cdot |A| - d - 1 \)

\( \Rightarrow \) every "grouping" loses one dimension.

**Def:** A \( d \)-manifold \( M \) with corners (modulo \( 1R^d_{+\infty} \))

**Thm:** \( C_A(M) \) is a \( d|A| \)-manifold with corners \( M \)

\[ \partial M = \prod_{A \in \text{CA}, |A| \geq 2} \{(p, c) : p \in C^0_A(M), c \in C_A(T_{p_\alpha}M)\} \]

**A complete proof would be Hell on Earth, and I'm not sure it was ever written. Sketch:**

**Blunders:**
1. Is a manifold w/ corners automatically a mod w/ boundary?
2. The discussion of the right tangent space on the left was lacking.
Thm 1. \( M \) compact \(\Rightarrow\) \( C_A(M) \) compact.

2. Singletons & doubletons:

3. \( B \subset A \Rightarrow \exists P_B : C_A(B) \to C_B(A) \). In particular,
\[ \exists \phi_{ij} : C_A(I^m) \to C_{\phi_{ij}}(I^n) \sim S^{n-1} \]

4. If \( P : M \to N \) is a smooth embedding,
\[ \exists f_* : C_A(M) \to C_A(N) \]

Skip section in bracket about \( C_D(M) \).

\[ C^0_D(M) := \{ p : A \to M : p(a_0) \neq p(a_1) \text{ whenever } a_0 \rightarrow a_1 \} \]

Definition 9. Write \( S^n = \mathbb{R}^n \setminus \{0\} \) and set \( \tilde{C}_A(\mathbb{R}^n) := \{ c \in \tilde{C}_A(\mathbb{R}^n) : p_\infty(c) = \infty \} \).

Theorem 10. \( \tilde{C}_A(\mathbb{R}^n) \) is a compact manifold with corners and the direction maps \( \phi_{ij} : \tilde{C}_A(\mathbb{R}^n) \to S^{n-1} \) remain well-defined.

Finally, given \( \gamma : S^1 \to \mathbb{R}^3 \) and disjoint finite sets \( A \) and \( B \), we set
\[ C^*_A,B := \{ (c',c) : c' \in C_A(S^1), c \in C_{A,B}(\mathbb{R}^3), \gamma(c') = p_A(c) \} \]
(and similarly \( C^*_D \) for appropriate graphs \( D \)). The obvious variants of the theorems remain valid.