Wednesday-2 AKT on 140115 (v1): The self-linking number and framings

1. \( l(s_1, s_2) = \sum_{x \in \mathbb{Z}} (-1)^x \)

2. \( \Phi: T^2 \to S^2 \) \( \Phi(s_1, s_2) = \frac{y(s_1) \times y(s_2)}{11} \)

\( l(y, z) = \int \Phi^* \omega, \quad \int S^2 = 1 \).

The linking number as a degree. Given \( \Phi: M^n \to N^n \)

between oriented manifolds,

\[
\deg \Phi = \frac{\Phi_*[M]}{\Phi_*[N]} = \int_M \Phi^* \omega = \sum_{x \in \mathbb{Z}} \deg_x \Phi
\]

is a homotopy invariant. \( l(y, z) = \deg \Phi_{y \times z} \)

On to \( S^2 \), following Maslovich’s paper (the emptiest discussion in mathematics — in two messy ways we will construct a knot invariant in \( \mathbb{Z}/2\mathbb{Z} \) …)

\[
\eta(x) := \frac{1}{4\pi} \int_{S^1} \Phi^* \omega = \sum_{x \in \mathbb{Z}} \Phi^* \omega
\]

\[
C_2(s') = \{(x, y): \frac{x + y}{x + y} = \frac{s'}{s'} \times y = x \}
\]

\[
\Phi(s', 0) := \lim_{x \to 0} \frac{y(x + x) - y(x)}{x + x} = \frac{y(x)}{x + x}
\]

\[
\Phi(s', 1) = \ldots = -y(x)
\]

\[
0 = \int_{C_2(S^1) \times I} \Phi^*_H \omega = \int_{\partial(C_2(S^1) \times I)} \Phi^*_H \omega = \eta_1 - \eta_0 + 2 \int_{S^1 \times I} \Phi^*_H \omega
\]