Wednesday-2 AKT on 140115 (v1): The self-linking number and framings

bring a tube & a laser pointer!

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1. $l(\gamma_1, \gamma_2) = \sum_{x \in \{\text{xings of } \gamma_1 \,\&\, \gamma_2\}} (-1)^x$

2. $\Phi : T^2 \to S^2 \quad \Phi(s_1, s_2) = \frac{\gamma_2(s_2) - \gamma_1(s_1)}{\|\quad\|}$

$l(\gamma_1, \gamma_2) = \int_T \Phi^* \omega, \quad \int_{S^2} \omega = 1.$

on board

3 The linking number as a degree: Given $\Phi : M^n \to N^n$ between oriented manifolds,

$$\deg \Phi = \frac{\Phi_*[M]}{\Phi_*[N]} = \int_M \Phi^* \omega_N = \sum_{x \in \Phi^{-1}(y)} \deg_x \Phi$$

is a homotopy invariant. $l(\gamma_1, \gamma_2) = \deg \Phi^{\gamma_1, \gamma_2}$

On to sl, following Moskovich's paper
(The emptiest discussion in Mathematics — in two messy ways we will construct a knot invariant in $\mathbb{Z}/\mathbb{Z}$ ...)

$$\eta(\gamma) := \frac{1}{4\pi} \int_{C_2(S^1)} \Phi^* \omega = \frac{1}{4\pi} \int_{\tilde{C}_2(S^1)} \Phi^* \omega$$

$$C_2(S^1) = \left\{(x, y) : \begin{matrix} x, y \in S^1 \\ x \neq y \end{matrix}\right\} = \underset{x}{S^1} \times (0,1)_z \quad y = x + z$$

$$\longrightarrow \tilde{C}_2(S^1) = S^1 \times [0, 1]$$

$$\Phi(x, 0) := \lim_{\epsilon \downarrow 0} \frac{\gamma(x+\epsilon) - \gamma(x)}{\|\quad\|} = \frac{\dot\gamma(x)}{\|\quad\|}$$

$$\Phi(x, 1) := \ldots \qquad = -\dot\gamma(x)$$

$$0 = \int_{\overline{C_2(S^1)} \times I} d\overline{\Phi_H^*}\omega = \int_{\partial(\overline{C_2(S^1)} \times I)} \overline{\Phi_H^*}\omega = \eta_1 - \eta_0 + 2 \int_{S^1 \times I} \overline{\Phi_H^*}\omega$$