HW7 returned, HW8 due, HW9 on web.

Last time: Given \( g \), \( W_g : A(g) \rightarrow U(g) \):

\( A \) is a "universal universal enveloping algebra"

Today's goals: The bi-algebra story, tangles

\( A \) is an algebra and so is \( U \). Is \( A \) a bi-algebra?

1. Define algebra \( (A, m, \varepsilon, \text{ diagrams}) \)
2. Define co-algebra \( (C, \Delta, \varepsilon, \text{ diagrams}) \)
3. Define bi-algebra.
4. \( A \) is a bi-algebra

\[ \exists \phi : A \rightarrow A \otimes A \text{ s.t. } \forall V_1, V_2 \in \mathcal{V} \]

\[ W_{V_1 V_2} = m_{\otimes} (W_{V_1} \otimes W_{V_2}) \circ \phi \]


6. Warning: Given \( g \)

\[ (A, m, \varepsilon) \Leftrightarrow (U(g), m, \varepsilon) \]

Questions:
1. What is \( \Delta \) in \( A \) language?
2. What is \( \varepsilon \) in \( U(g) \) language?

Tangles [knots]

1. Delete an edge
2. Double an edge
3. Planar algebra

Question: What is gray? What's "an expansion"?