Graph Cohomology and Configuration Space Integrals

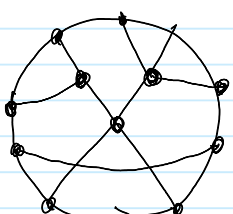
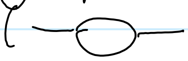
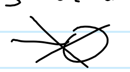
March-04-14 3:39 PM

WT = Wishful Thinking := The boundary of configuration spaces C_D^γ consists only of the principal faces, in which just two points cluster together.

Theorem (WT) The following diagram commutes

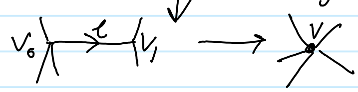
$$\begin{array}{ccccccccl}
\overline{\mathcal{D}}_n^\bullet : & \cdots \longrightarrow & \overline{\mathcal{D}}_n^m & \xrightarrow{d} & \overline{\mathcal{D}}_n^{m+1} & \longrightarrow \cdots & & & \text{"The graph complex"} \\
& & \downarrow I & & \downarrow I & & & & \\
\Omega^\bullet : & \cdots \longrightarrow & \Omega^m(\Gamma) & \xrightarrow{d} & \Omega^{m+1}(\Gamma) & \longrightarrow \cdots & & & \text{"Knot space cohomology"}
\end{array}$$

where Γ is the space of smooth $\gamma: S^1 \hookrightarrow \mathbb{R}^3$, $\Omega^\bullet(\Gamma)$ is the de-Rham complex of Γ,

$\overline{\mathcal{D}}_n^m = \Bigg\langle D =$  connected $:$
$m = m(D) = \sum_{v \in V(D)} (\deg v - 3) = 2|E| - 3|V|$
$n = n(D) = -\chi(D) = |E| - |V|$
directed internal edges / $\rightarrow + \leftarrow = 0$
ordered vertices / alternating rep.
double internal edges allowed ()
but no bubbles  $\Bigg\rangle$

$d: \overline{\mathcal{D}}_n^m \to \overline{\mathcal{D}}_n^{m+1}$ is $dD := \sum_{\substack{e \in E(D) \text{ non-chord} \\ \text{not part of a multiple edge}}} \pm D/e$

1. signed remove v_0
2. signed remove v_1
3. signed insert v



$I: \overline{\mathcal{D}}_n^m \to \Omega^m(\Gamma)$ is $\pi_* \Phi_D^* \omega^{E_i(D)}$, where

$$\begin{array}{ccccc}
C_D^\gamma & \longrightarrow & T & \xrightarrow{\Phi_D} & (S^2)^{E_i(D)} \\
& & \downarrow \pi & & \\
& & \Gamma & &
\end{array}$$

Points to note: 1. signs work out! 2. Chords work out! 3. Double edges work out!