Friday-2 AKT on 140117: Scratch

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$$(2\pi)^{\frac{n+1}{2}} = \int_{\mathbb{R}^{n+1}} e^{-x^2/2} = \int_0^\infty V_n\, r^n\, e^{-r^2/2}\, dr$$

$$V_n = (2\pi)^{\frac{n+1}{2}} / I_n$$

$$I_n := \int_0^\infty r^n e^{-r^2/2}\, dr$$

$$I_0 = \frac{1}{2}\sqrt{2\pi}$$

$$I_1 = -e^{-r^2/2}\Big|_0^\infty = 1$$

$$= \int_0^\infty \underbrace{r^{n-1}}_{f}\, \underbrace{r e^{-r^2/2}}_{g'}\, dr = fg\Big|_0^\infty + \int_0^\infty (n-1) r^{n-2} e^{-r^2/2}\, dr$$

$$= (n-1) I_{n-2}$$

$$I_2 = I_0 = \frac{1}{2}\sqrt{2\pi} \qquad V_2 = \frac{(2\pi)^{3/2}}{\frac{1}{2}(2\pi)^{1/2}} = 4\pi \quad \checkmark$$

$$V_1 = 2\pi \quad \checkmark$$

$$V_0 = (2\pi)^{1/2} \Big/ \frac{1}{2}\sqrt{2\pi} = 2 \quad \checkmark$$