14-1350-AKT Homework Assignment 8

March-03-14 8:37 AM

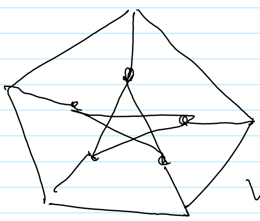
This assignment is due in class on Monday, March 24, 2014.

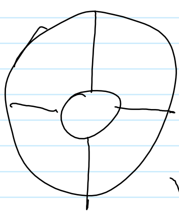
Question #1. Read HPMOR at hpmor.com and explain, in your words, how time travel would allow for the solution in polynomial time of arbitrary NP problems.

Just kidding. The solution to this problem is indeed at said URL, but it is not related to our class. Now for real — do just one of Question #1 & Question #3 below, yet make sure that you read and fully understand also the other one.

Question #1. Note that the definition of $W_{\mathfrak{g}}$ extends to diagrams that do not have a skeleton. In fact, in that case, no representation of $\mathfrak{g}$ is needed.

Let D by a "Vacuum diagram" — a connected (thanks, Huan), diagram that has no skeleton. Let v be the number of vertices in D. For example:

$D_1 =$  $v=10$

$D_2 =$  $v=8$

Prove that $W_{gl(N)}(D)$ is a polynomial in N of degree at most $\frac{v}{2}+2$, and that

The coefficient of $N^{v/2+2}$ in $W_{gl(N)}(D)$ is non-zero iff D can be embedded in the plane without self intersections.
(Above, D_1 cannot be embedded, but D_2 can).

Question 2 There is no question 2.

Question 3 Recall that in the language of our Friday classes,

$$A^g = g^{-1}Ag + g^{-1}dg .$$

1. Show that if δg is an infinitesimal gauge-transformation (namely, $\delta g : M \to \mathfrak{g}(=M_{n\times n})$), then

$$\frac{\delta A}{\delta g} := A^{I+\delta g} - A = d\delta g + [A, \delta g]$$

2. With δA an infinitesimal member of $\Omega^1(M)$ (M a 3D closed manifold (compact, $\partial M = \emptyset$)), show that

thanks, Travis — this trace was missing in an earlier version of this assignment. Thanks, Chris Adkins, for noting.

$$\frac{\delta CS(A)}{\delta A} = 2\int_{M^3} tr(F_A \wedge \delta A),$$

where $F_A = dA + A \wedge A$ is the "curvature" of A

3. Show that $CS(A)$ is invariant under infinitesimal gauge transformations.