14-1100 Sep 11, hour 3: Kernels, normal subgroups, quotients

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$$\sigma_{8,j}(\sigma_{4,j_4}M_5) \stackrel{1}{=} (\sigma_{8,j}\sigma_{4,j_4})M_5 \stackrel{2}{\subset} M_4M_5$$

$$\stackrel{3}{=} \sigma_{4,j_4'}(M_5M_5) \stackrel{4}{\subset} \sigma_{4,j_4'}M_5 \subset M_4$$

on board.

Read Along: Selick's notes 1.1, 1.2.1, 1.4; Lang's book I 1-3.

Very quickly: groups, uniqueness of 1, $^{-1}$, $(ab)^{-1} = b^{-1}a^{-1}$

order of an element.

Group homomorphisms, "The category of groups"

The group Aut(G)

Conjugation: $g^h = h^{-1}gh = C_h(g)$ $\quad (g_1g_2)^h = g_1^h g_2^h \quad g^{h_1h_2} = (g^{h_1})^{h_2}$

$h \mapsto C_h$ is an anti-homomorphism $G \longrightarrow Aut(G)$

Images, kernels, subgroups.

Example: S_3 is an image of S_4, but not a kernel.

Normal subgroups, kernels are normal.

<u>Question</u> Is every normal subgroup the kernel of a homomorphism? Given $N \triangleleft G$, can we find a surjective homomorphism $\phi: G \to H$, with $\ker\phi = N$?

Set theoretic aside: Surjections are the same as equivalence relations.

(def'n, explanation ...)

done fine.

Sol'n Suppose we had ϕ, consider the resulting equiv:

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$g_1 \sim g_1 n$ or $g_1 \sim g_2$ iff $g_1^{-1} g_2 \in N$.

Let $H = G/\sim = \{[g]\}$ where $[g] = gN$

with $\phi : G \to H$ being $\phi(g) = [g]$

define $[g_1][g_2] = [g_1 g_2]$ (well defined?)

$[g]^{-1} = [g^{-1}]$

Claim $H = G/\sim$ is a group & ϕ is a morphism whose kernel is N ... we write $H = G/N$.

Theorem (The First Isomorphism Theorem) Given any morphism $\phi : G \to H$, $G/\ker\phi \cong \operatorname{im}\phi$.